Nuclear mean field theories and collective phenomena

L. Próchniak

Maria Curie-Skłodowska University, Lublin

Outline

Mean field theories

Collective states, Adiabatic Time Dependent Hartree-Fock-Bogolyubov, Generator Coordinate Method

Quadrupole excitations, Bohr Hamiltonian

Examples

General HFB theory

Variational method. Product states as test functions

$$\hat{H}_{\text{micr}} = \sum_{\mu,\nu} K_{\mu\nu} d^+_\mu d_\nu + \frac{1}{4} \sum_{\mu,\nu,\alpha,\beta} \tilde{V}_{\mu\nu\alpha\beta} d^+_\mu d^+_\nu d_\beta d_\alpha$$

Product states: BCS type states, Slater determinants as a special case

$$\alpha^+_\mu = \sum_\nu U_{\nu\mu} d^+_\nu + \sum_\nu V_{\nu\mu} d_\nu = u_\mu c^+_\mu + s^*_\mu v_{\bar{\mu}} c_{\bar{\mu}}$$

$$\Psi_{\text{BCS}} = \prod_{\mu>0} (u_\mu + s_\mu v_{\bar{\mu}} c^+_{\bar{\mu}} c^+_\mu) \left|0\right\rangle$$

$$\mathcal{R} = \begin{pmatrix} V^* V^T & V^* U^T \\ U^* V^T & U^* U^T \end{pmatrix} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1-\rho^* \end{pmatrix} = \begin{pmatrix} \langle\Psi| d^+_\nu d_\mu |\Psi\rangle & \langle\Psi| d_\nu d_\mu |\Psi\rangle \\ \langle\Psi| d^+_\nu d^+_\mu |\Psi\rangle & \langle\Psi| d_\nu d^+_\mu |\Psi\rangle \end{pmatrix}$$

Canonical basis

$$\rho_{\mu\nu} = v^2_\mu \delta_{\mu\nu} \; \kappa_{\mu\nu} = s_{\bar{\mu}} u_\mu v_\mu \delta_{\bar{\mu}\nu}$$

General HFB theory, cont.

$$[\mathcal{W}(\mathcal{R}), \mathcal{R}] = 0$$

$$\mathcal{W}(\mathcal{R}) = \begin{pmatrix} K + \mathit{\Gamma} - \lambda I & \mathit{\Delta} \\ -\mathit{\Delta}^* & -K^* - \mathit{\Gamma}^* + \lambda I \end{pmatrix} = \begin{pmatrix} h_0 - \lambda I & \mathit{\Delta} \\ -\mathit{\Delta}^* & -h_0 + \lambda I \end{pmatrix}$$

$$\mathit{\Gamma}_{\mu\nu} = \sum_{\mu',\nu'} \tilde{V}_{\mu\mu'\nu\nu'} \rho_{\nu'\mu'} + \text{ rear. terms} = \frac{\partial}{\partial \rho_{\mu\nu}} E[\mathcal{R}]$$

$$\mathit{\Delta}_{\mu\nu} = \frac{1}{2} \sum_{\mu',\nu'} \tilde{V}_{\mu\nu\mu'\nu'} \kappa_{\mu'\nu'} + \text{ rear. terms} = \frac{\partial}{\partial \kappa_{\mu\nu}} E[\mathcal{R}]$$

$$E[\mathcal{R}] = \langle \Psi | \hat{H}_{\text{micr}} | \Psi \rangle$$

Skyrme interaction

Momentum space

$$\langle \boldsymbol{k}|\, V\, |\boldsymbol{k}'\rangle = \tilde{t}_0(1 + x_0 P_\sigma) + \frac{1}{2}\tilde{t}_1(\boldsymbol{k}^2 + \boldsymbol{k}'^2) + \tilde{t}_2 \boldsymbol{k}\boldsymbol{k}' + i\tilde{W}_0(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot(\boldsymbol{k}\times\boldsymbol{k}') + v_{123}$$

Kernel of an integral operator $\langle f(1,2)|V_S|g(1,2)\rangle$

$$\begin{aligned} V_S \quad = \quad & t_0(1 + x_0 P_\sigma)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) + \frac{1}{2}t_1(\boldsymbol{k}^2\delta(\boldsymbol{r}_1-\boldsymbol{r}_2) + \delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}'^2) + t_2\boldsymbol{k}\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}' + \\ & + iW_0(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot(\boldsymbol{k}\times\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}') + \\ & + \tilde{t}_3\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\delta(\boldsymbol{r}_2-\boldsymbol{r}_3) \longrightarrow \frac{1}{6}t_3(1 + P_\sigma)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\rho^\alpha((\boldsymbol{r}_1 + \boldsymbol{r}_2)/2) \end{aligned}$$

$$\boldsymbol{k}' = \frac{1}{2i}(\overrightarrow{\boldsymbol{\nabla}}_1 - \overrightarrow{\boldsymbol{\nabla}}_2), \quad \boldsymbol{k} = -\frac{1}{2i}(\overleftarrow{\boldsymbol{\nabla}}_1 - \overleftarrow{\boldsymbol{\nabla}}_2),$$

plus Coulomb for protons

Gogny interaction

$$
\begin{aligned}
V_G \quad = \quad & \sum_{j=1,2} \exp(|\boldsymbol{r}_1 - \boldsymbol{r}_2|^2/{a_j}^2)(W_j + B_j P_\sigma - H_j P_\tau - M_j P_\sigma P_\tau) + \\
& + iW_{G0}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{k} \times \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{k}') + \\
& + t'_{G3}(1 + P_\sigma)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\rho^\alpha((\boldsymbol{r}_1 + \boldsymbol{r}_2)/2)
\end{aligned}
$$

$$a_1 = 0.7 \text{ fm}, \quad a_2 = 0.2 \text{ fm}, \quad \alpha = 1/3$$

plus Coulomb

Relativistic Mean Field

One of numerous versions. Dirac equation with the self-consistent potential.

$$(\boldsymbol{\alpha} \cdot \boldsymbol{p} + V(\boldsymbol{r}) + \beta(M + S(\boldsymbol{r})))\psi_j(\boldsymbol{r}) = \epsilon_j \psi_j(\boldsymbol{r})$$

$$V(\boldsymbol{r}) = g_\omega \omega^0(\boldsymbol{r}) + g_\rho \tau_3 \rho^0(\boldsymbol{r}) + e\tfrac{1-\tau_3}{2} A^0(\boldsymbol{r})$$
$$S(\boldsymbol{r}) = g_\sigma \sigma(\boldsymbol{r})$$

$$(-\Delta + {m_\sigma}^2)\sigma(\boldsymbol{r}) + g_2\sigma^2(\boldsymbol{r}) + g_3\sigma^3(\boldsymbol{r}) = -g_\sigma \rho_s(\boldsymbol{r})$$
$$(-\Delta + {m_\omega}^2)\omega^0(\boldsymbol{r}) = g_\omega \rho_v(\boldsymbol{r})$$
$$(-\Delta + {m_\rho}^2)\rho^0(\boldsymbol{r}) = g_\rho \rho_3(\boldsymbol{r})$$
$$-\Delta A^0(\boldsymbol{r}) = e\rho_c(\boldsymbol{r})$$

$$\rho_s(\boldsymbol{r}) = \textstyle\sum_i \bar{\psi}_i(\boldsymbol{r})\psi_i(\boldsymbol{r}) \qquad \rho_v(\boldsymbol{r}) = \sum_i \psi_i^\dagger(\boldsymbol{r})\psi_i(\boldsymbol{r})$$
$$\rho_3(\boldsymbol{r}) = \textstyle\sum_i \psi_i^\dagger(\boldsymbol{r})\tau_3\psi_i(\boldsymbol{r}) \qquad \rho_c(\boldsymbol{r}) = \sum_i \psi_i^\dagger(\boldsymbol{r})\frac{1-\tau_3}{2}\psi_i(\boldsymbol{r}).$$

Pairing interaction

Constant G (seniority force)

$$G \sum_k d_k^+ d_{\bar{k}}^+ d_k d_{\bar{k}}$$

δ interaction:

$$V_0 \delta(\boldsymbol{r} - \boldsymbol{r}'),$$

$$V_0(\rho(\boldsymbol{r}))\delta(\boldsymbol{r} - \boldsymbol{r}'), \text{ e.g. } V_0(\rho) = 1 - \rho(\boldsymbol{r})/\rho_0$$

Gogny type interaction (only ,,Gaussian" part)

Only p-p and n-n pairing

Applications

Nuclear ground state properties (binding energies, radii, static deformation, fission barriers), giant resonances, nuclear matter properties

Recent review papers

M. Bender, P.-H. Heenen and P.-G. Reinhard, *Self-consistent mean-field models for nuclear structure*, Rev.Mod.Phys. **75** (2003) 121.

J.R. Stone and P.-G. Reinhard, *The Skyrme interaction in finite nuclei and nuclear matter*, Prog. Part. Nucl. Phys. **58** (2007) 587.

T. Niksic , D. Vretenar and P. Ring, *Relativistic nuclear energy density functionals: Mean-field and beyond*, Prog. Part. Nucl. Phys. in press.

Collective states

Cannot be properly described by single-particle excitations

Mean field is fixed, occupation numbers are changing,
e.g. RPA, giant resonances (not discussed)

Mean field is changing, occupation numbers are fixed,
e.g. change of a nuclear ,,shape”

Main methods: ATDHFB and GCM (plus GOA):
a set of product states parametrized by several (collective) variables

$$\downarrow$$

Schroedinger type equation in the collective space

Adiabatic approximation of the Time Dependent HFB theory

Time dependent HFB

$$i\hbar\dot{\mathcal{R}} = [\mathcal{W}(\mathcal{R}), \mathcal{R}]$$

Adiabatic approximation

$$\mathcal{R} = \exp(i\chi(t))\mathcal{R}_0(t)\exp(-i\chi(t)) = \mathcal{R}_0 + \mathcal{R}_1 + \mathcal{R}_2 + \dots$$

$$\mathcal{W} = \mathcal{W}_0 + \mathcal{W}_1 + \mathcal{W}_2 + \dots$$

$$[\mathcal{W}_0, \mathcal{R}_0] \approx 0 \quad \text{(of the second order)}$$

$$i\hbar\dot{\mathcal{R}}_0 = [\mathcal{W}_0, \mathcal{R}_1] + [\mathcal{W}_1, \mathcal{R}_0]$$

Several collective variables

Collective variables α. $\mathcal{R}(t) = \mathcal{R}(\alpha(t))$

$$i\hbar\dot{\alpha}_k \frac{\partial \mathcal{R}_0}{\partial \alpha_k} = [\mathcal{W}_0, \mathcal{R}_1^k] + [\mathcal{W}_1(\mathcal{R}_1^k), \mathcal{R}_0], \quad k = 1, ..., n$$

$$\langle \Psi | H_{\mathrm{micr}} | \Psi \rangle = T_{\mathrm{cl}} + V_{\mathrm{cl}} = H_{\mathrm{cl}}$$

$$V_{\mathrm{cl}} = \langle \Psi_0(\alpha) | H_{\mathrm{micr}} | \Psi_0(\alpha) \rangle$$

$$T_{\mathrm{cl}} = \frac{1}{2} \sum_{k,j} B_{kj}(\alpha) \dot{\alpha}_k \dot{\alpha}_j$$

Mass parameters (inertial functions)

$$B_{kj}(\alpha) = \frac{i\hbar}{2\dot{\alpha}_j} \mathrm{Tr}_{2d}(\mathcal{R}_1^j \, [\frac{\partial \mathcal{R}_0}{\partial \alpha_k}, \mathcal{R}_0])$$

Cranking Approximation

Assumption

$$[\mathcal{W}_1(\mathcal{R}_1^k), \mathcal{R}_0] \approx 0$$

$$B_{kj} = \frac{\hbar^2}{2} \sum_{\mu,\nu} \frac{f_{j,\mu\nu} f^*_{k,\mu\nu} + f^*_{j,\mu\nu} f_{k,\mu\nu}}{(E_\mu + E_\nu)} \; .$$

$$f_{k,\mu\nu} = s_\nu (\partial_k \rho)_{\mu\bar{\nu}} (u_\mu v_\nu + v_\mu u_\nu) + (\partial_k \kappa)_{\mu\nu} (u_\mu u_\nu - v_\mu v_\nu)$$

$f_{k,\mu\nu} = \langle \Psi_0 | a_\nu a_\mu | \partial_k \Psi_0 \rangle, \quad a_\mu$ — quasiparticle operators

$$f_{k,\mu\nu} = -\frac{1}{E_\mu + E_\nu} [s_\nu (\partial_k h_0)_{\mu\bar{\nu}} (u_\mu v_\nu + v_\mu u_\nu) + (\partial_k \Delta)_{\mu\nu} (u_\mu u_\nu - v_\mu v_\nu)]$$

Requantization

Classical expression $T_{\rm cl} + V_{\rm cl} \to H_{\rm quant}$

$$T_{\rm cl} = \frac{1}{2}\sum_{k,j} B_{kj}(\boldsymbol{\alpha})\dot{\alpha}_k\dot{\alpha}_j$$

Laplace-Beltrami operator

$$T_{\rm quant} = -\frac{\hbar^2}{2}\frac{1}{\sqrt{\det B}}\sum_{k,j}\frac{\partial}{\partial\alpha_k}\sqrt{\det B}\,(B^{-1})_{kj}\frac{\partial}{\partial\alpha_j}$$

Volume element $\sqrt{\det B}\, d\alpha_1 \ldots d\alpha_n$

Generator Coordinate Method +GOA

Once again the variational principle. Test functions $\int d\boldsymbol{\alpha} f(\boldsymbol{\alpha})\Psi(\boldsymbol{\alpha})$

Gaussian Overlap Approximation

$$\langle\Psi(\boldsymbol{\alpha}'')|\Psi(\boldsymbol{\alpha}')\rangle = \exp(-\sum_{k,j} g_{kj}(\boldsymbol{\alpha})(\alpha''_k - \alpha'_k)(\alpha''_j - \alpha'_j)/2), \quad \boldsymbol{\alpha} = (\boldsymbol{\alpha}'' + \boldsymbol{\alpha}')/2$$

$$H_{\mathrm{GCM}} f(\boldsymbol{\alpha}) = E f(\boldsymbol{\alpha})$$

$$H_{\mathrm{GCM}} = T_{\mathrm{GCM}} + V_{\mathrm{GCM}}$$

$$h(\boldsymbol{\alpha}'', \boldsymbol{\alpha}') = \langle\Psi(\boldsymbol{\alpha}'')|H_{\mathrm{micr}}|\Psi(\boldsymbol{\alpha}')\rangle / \langle\Psi(\boldsymbol{\alpha}'')|\Psi(\boldsymbol{\alpha}')\rangle$$

$$g_{kj}(\boldsymbol{\alpha}) = \langle\partial_{\alpha_k}\Psi(\boldsymbol{\alpha})|\partial_{\alpha_j}\Psi(\boldsymbol{\alpha})\rangle$$

$$T_{\text{GCM}} = -\frac{\hbar^2}{2}\frac{1}{\sqrt{\det g}}\sum_{k,j}\frac{\partial}{\partial \alpha_k}\sqrt{\det g}\,(B^{-1}_{\text{GCM}})^{kj}\frac{\partial}{\partial \alpha_j}\ .$$

$$(B^{-1}_{\text{GCM}})^{kj} = \frac{1}{2\hbar^2}(\text{Re}\,h_{12} - \text{Re}\,h_{11})^{kj}$$

$$h_{11,mn} = D_{\alpha''_m}D_{\alpha''_n}h(\alpha'',\alpha')|_{\alpha'=\alpha''=\alpha}$$
$$h_{12,mn} = D_{\alpha''_m}D_{\alpha'_n}h(\alpha'',\alpha')|_{\alpha'=\alpha''=\alpha}$$

D — covariant derivative
Potential energy

$$V_{\text{GCM}} = V_{\text{cl}}(\alpha) + V_{\text{ZPE}}(\alpha)$$

$$V_{\text{cl}}(\alpha) = \langle\Psi(\alpha)|H_{\text{micr}}|\Psi(\alpha)\rangle$$
$$V_{\text{ZPE}}(\alpha) = -\frac{\hbar^2}{2}\sum_{k,j}g^{kj}(B^{-1}_{\text{GCM}})_{kj} - \frac{1}{8}\sum_{k,j}g^{kj}D_{\alpha_k}D_{\alpha_j}V_{\text{cl}}(\alpha)\ .$$

Quadrupole variables

1. Quadrupole mass tensor $Q_{2\mu} = \langle\Psi| \sum_i r_i^2 Y_{2\mu}(i)\Psi\rangle$
2. Nuclear surface $r(\alpha) = r_0(1 + \sum_\mu \alpha_\mu^* Y_{2\mu})$
3. Ellipsoidal shape (e.g. of a nucleus or one-particle potential) $\sum_{k,j} w_{kj}x_k x_j = 1$

(...)

Principal axes system (intrinsic system)
Spherical tensors (α or Q)

$$\{\alpha_\mu\} \xrightarrow{R(\Omega)} \{\tilde{\alpha}_0, \tilde{\alpha}_1 = \tilde{\alpha}_{-1} = 0, \tilde{\alpha}_2 = \tilde{\alpha}_{-2}\}$$

Cartesian case (ellipsoid)

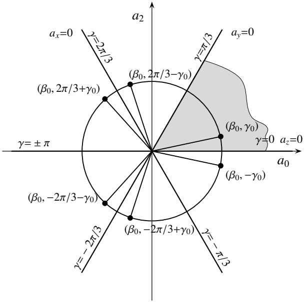
$$\sum_{k,j} w_{kj}x_k x_j = 1 \xrightarrow{R(\Omega)} \sum_k \tilde{w}_k x_k^2 = 1$$

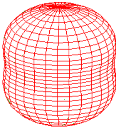
Quadrupole variables, cont.

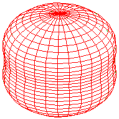
Deformation variables β, γ

$$\begin{aligned} \tilde{\alpha}_0 &= \beta \cos\gamma, \\ \tilde{\alpha}_2 = \tilde{\alpha}_{-2} &= \beta \sin\gamma / \sqrt{2} \end{aligned}$$

LAB $\longleftrightarrow$ INT: $\alpha_\mu(Q_{2\mu}) \longleftrightarrow (\beta, \gamma,$ Euler angles $\Omega)$







Quadrupole variables in the mean field approach

Deformation variables

$$\beta\cos\gamma = cq_0 = c\langle\Psi|Q_0|\Psi\rangle = \langle\Psi|\sum_{i=1}^{A}(3z_i^2 - r_i^2)|\Psi\rangle$$
$$\beta\sin\gamma = cq_2 = c\langle\Psi|Q_2|\Psi\rangle = \langle\Psi|\sum_{i=1}^{A}\sqrt{3}(x_i^2 - y_i^2)|\Psi\rangle; \quad c = \sqrt{\pi/5}/A\overline{r^2}$$

HFB with constraints

$$\delta\langle\Psi|H_{\text{micr}} - \lambda_0 Q_0 - \lambda_2 Q_2|\Psi\rangle = 0$$
$$\langle\Psi|Q_0|\Psi\rangle = q_0, \quad \langle\Psi|Q_2|\Psi\rangle = q_2$$

Mass parameters

$$B_{q_i q_j} = \hbar^2 (S_{(1)}^{-1} S_{(3)} S_{(1)}^{-1})_{ij}$$

$$(S_{(n)})_{ij} = \sum_{\mu,\nu} \frac{\langle\mu|Q_i|\bar{\nu}\rangle\langle\bar{\nu}|Q_j|\mu\rangle}{(E_\mu + E_\nu)^n}(u_\mu v_\nu + u_\nu v_\mu)^2$$

Moments of inertia

$$J_k = \hbar^2 \sum_{\mu,\nu} \frac{|\langle\nu|j_k|\bar{\mu}\rangle|^2 (u_\mu v_\nu - u_\nu v_\mu)^2}{(E_\mu + E_\nu)}$$

Kinetic energy in the intrinsic frame

Five variables β, γ, Ω.

Mass parameters matrix

$$B = \begin{pmatrix} B_{\mathrm{vib}} & 0 \\ 0 & B_{\mathrm{rot}} \end{pmatrix}$$

Angular momentum components instead of ∂_{Ω_k}

$$B_{\mathrm{vib}} = \begin{pmatrix} B_{\beta\beta} & \beta B_{\beta\gamma} \\ \beta B_{\beta\gamma} & \beta^2 B_{\gamma\gamma} \end{pmatrix}$$

$$B_{\mathrm{rot}} = \begin{pmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{pmatrix}$$

$$J_k = 4\beta^2 B_k(\beta,\gamma)\sin^2(\gamma - 2\pi k/3)$$

Quantum Hamiltonian in the intrinsic frame

(General) Bohr Hamiltonian

$$H_{\text{Bohr}} = T_{\text{vib}} + T_{\text{rot}} + V$$

$$T_{\text{vib}} = -\frac{1}{2\sqrt{wr}}\left\{\frac{1}{\beta^4}\left[\partial_\beta\left(\beta^4\sqrt{\frac{r}{w}}B_{\gamma\gamma}\right)\partial_\beta - \partial_\beta\left(\beta^3\sqrt{\frac{r}{w}}B_{\beta\gamma}\right)\partial_\gamma\right]+\right.$$

$$\left.+\frac{1}{\beta\sin 3\gamma}\left[-\partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\gamma}\right)\partial_\beta + \frac{1}{\beta}\partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\beta}\right)\partial_\gamma\right]\right\}$$

$$T_{\text{rot}} = \frac{1}{2}\sum_{k=1}^{3} I_k^2(\Omega)/J_k; \quad J_k = 4B_k(\beta,\gamma)\beta^2\sin^2(\gamma - 2\pi k/3)$$

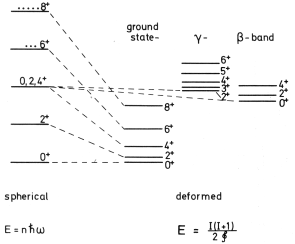
$$w = B_{\beta\beta}B_{\gamma\gamma} - B_{\beta\gamma}^2; \quad r = B_x B_y B_z$$

Energy levels, B(E2) transition probabilities

Special cases

Simple kinetic energy, $B_{\beta\beta} = B_{\gamma\gamma} = B_k = B$, $B_{\beta\gamma} = 0$

Harmonic oscillator, $V \sim \beta^2$



Other analytical solutions, dynamical symmetries, critical symmetries, connections with other models

First 2^+ state. Experimental data

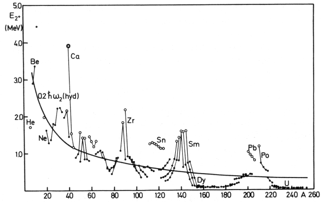


Figure 1.7. The energy of the first 2^+ state in even–even nuclei. The nuclei with closed neutron or proton shells are marked by open circles. (From [NN 65].)

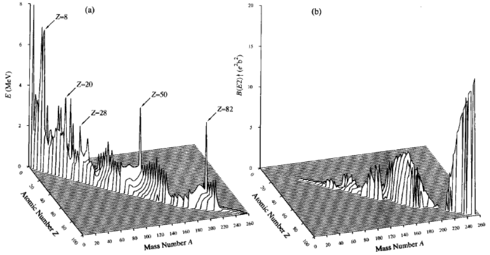
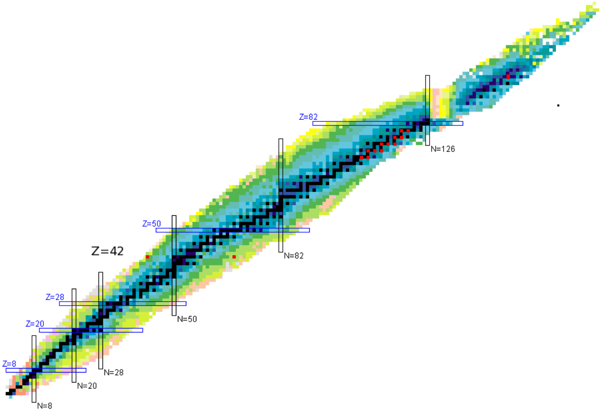


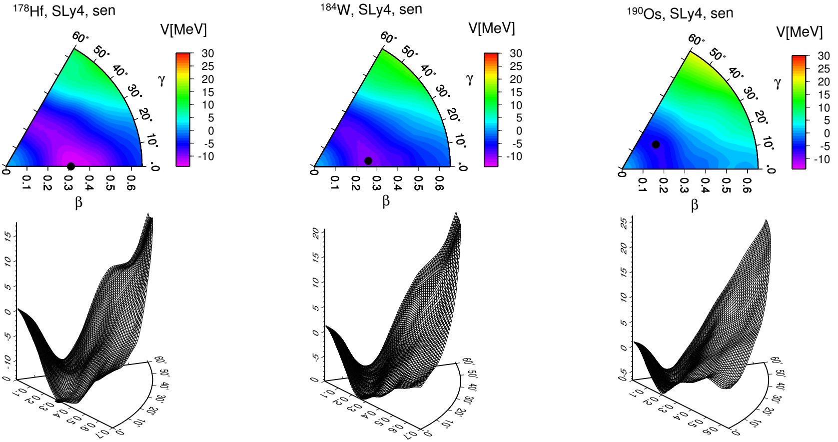
FIG. A. (a) Energies of the first-excited 2^+ states in even–even nuclei and (b) their corresponding reduced electric quadrupole transition probability $B(E2)\uparrow$ values. This figure is based on the adopted values of Table I.

Examples

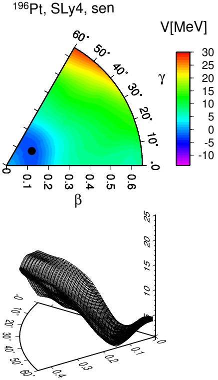
1. From well deformed Hf to almost spherical Hg: $^{178,180}_{72}$Hf, $^{182-186}_{74}$W, $^{188-192}_{76}$Os, $^{194,196}_{78}$Pt, $^{198,200}_{80}$Hg
2. Molybdenum isotopes, $^{84-110}$Mo
3. ^{240}Pu

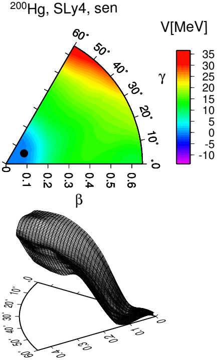


^{178}Hf — ^{200}Hg. Potential energy surfaces

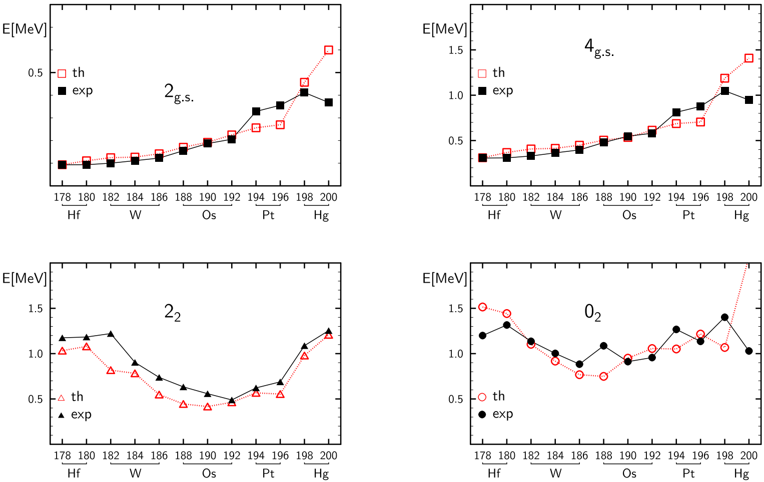


^{178}Hf — ^{200}Hg. Potential energy surfaces, cont.

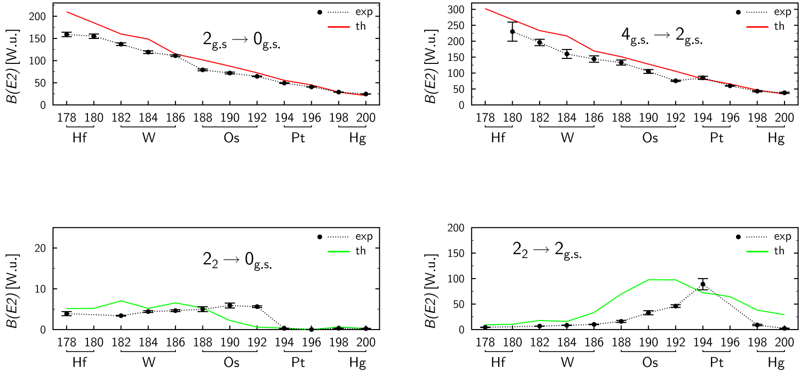




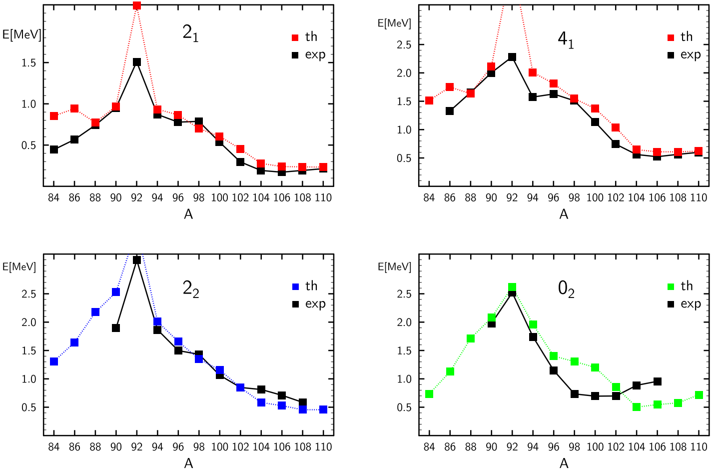
^{178}Hf — ^{200}Hg. Energy levels



^{178}Hf — ^{200}Hg. B(E2) transition probabilities

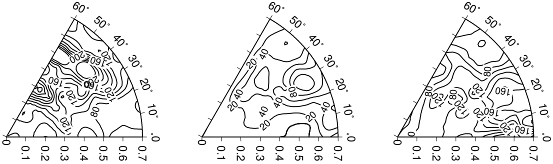


$^{84-110}$Mo isotopes. Energy levels

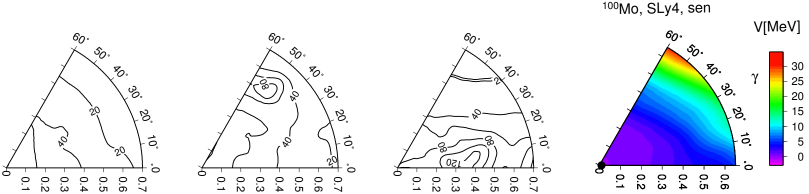


^{100}Mo. Potential energy, mass parameters

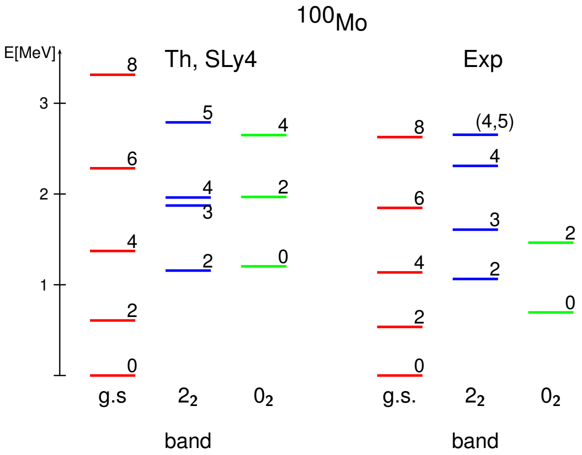
Mass parameters $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$



Parameters B_k, $k = x, y, z$, potential energy

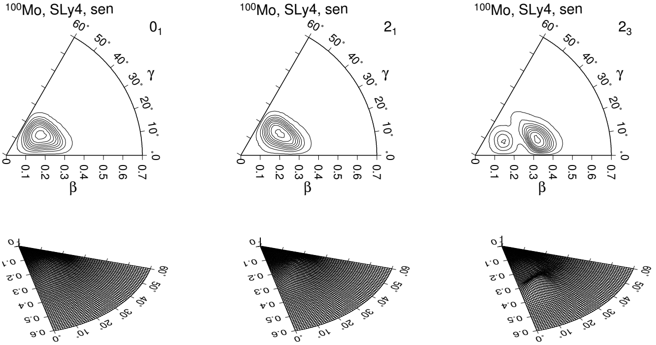


^{100}Mo. Energy levels



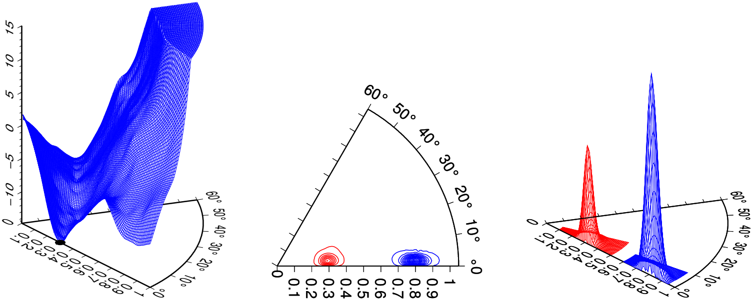
^{100}Mo. Collective wave functions

Probability density $|\Phi_{\rm coll}|^2 d\tau = |\Phi_{\rm coll}|^2 \beta^4 |\sin 3\gamma| \tilde{w}(\beta,\gamma)$



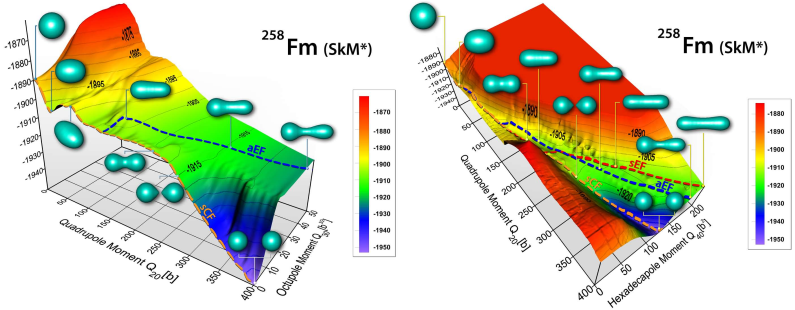
^{240}Pu. Collective states in the second minimum of the potential

Probability density for the normal and superdeformed ground state



Fission

WKB , fission paths, lifetimes
Different variables, axial shapes



A.Staszczak, A.Baran, J.Dobaczewski, W.Nazarewicz, Phys.Rev. C **80**, 014309 (2009)