Group D Nuclear Reactions - Theory

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International Workshop on Acceleration and Applications of Heavy Ions

Plan of the talk

1. Introduction
   - Interactions of Nuclei: Scattering and Reaction
   - Cross-Sections
2. Theoretical Models
   - Elastic Scattering
   - Optical Model
   - Coupled Channels Model
   - α cluster Model for Transfer
3. Data Analysis with Fresco
   - Fresco and Sfresco
   - Data Analysis

Interactions of Nuclei

- Scattering
  - Elastic: Nuclei *and* energy remain the same

$$a + A \Rightarrow a + A, \qquad Q = 0$$

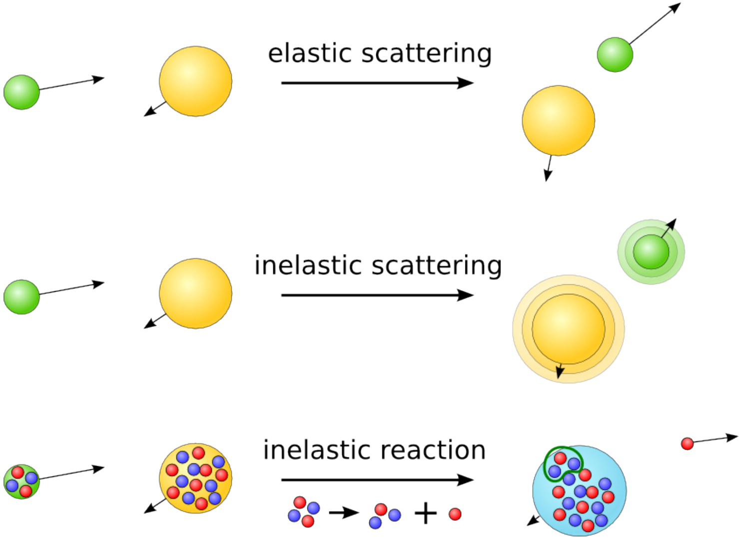
$$(Q = (m_{\text{initial}} - m_{\text{final}})c^2)$$

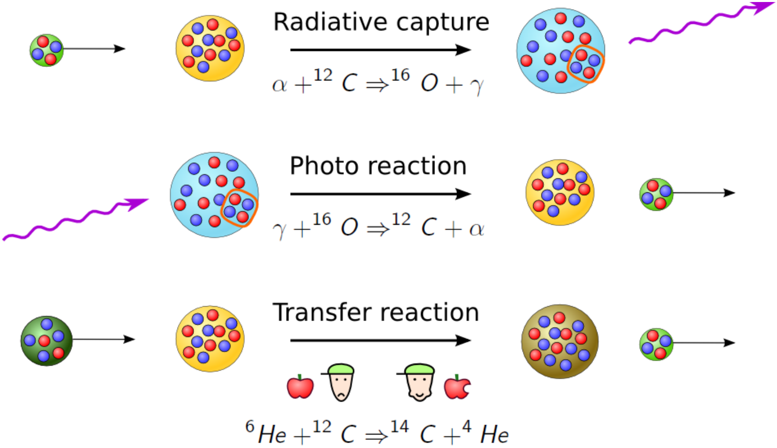
  - Inelastic

$$a + A \Rightarrow a^* + A^*, \qquad Q < 0$$

- Inelastic Reaction

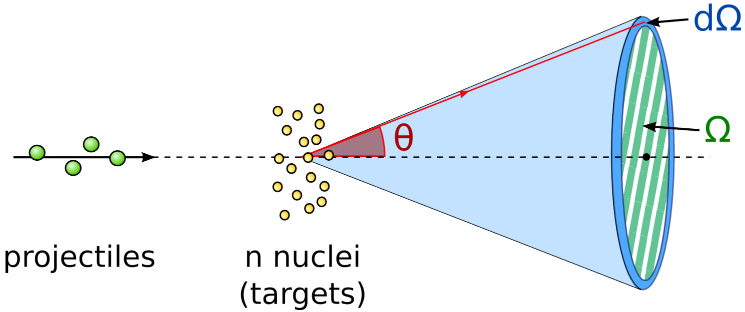
$$a + A \Rightarrow b + B$$


elastic scattering
inelastic scattering
inelastic reaction


Radiative capture
$\alpha + ^{12}C \Rightarrow ^{16}O + \gamma$
Photo reaction
$\gamma + ^{16}O \Rightarrow ^{12}C + \alpha$
Transfer reaction
$^{6}He + ^{12}C \Rightarrow ^{14}C + ^{4}He$

Observable: Cross-section

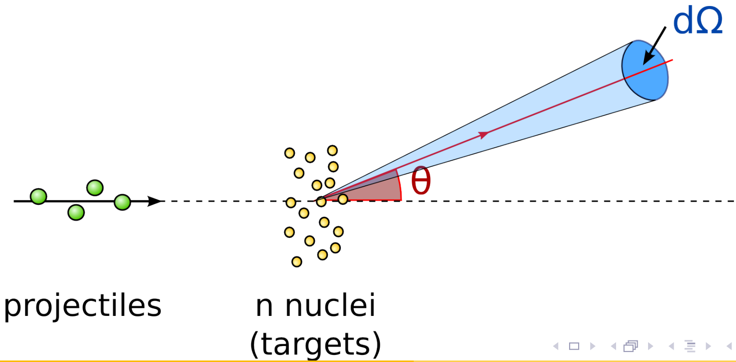
$$\sigma = \frac{\text{detected particles per unit time}}{\text{projectiles current} \times \text{count of target nuclei}}$$



Differential Cross-Section

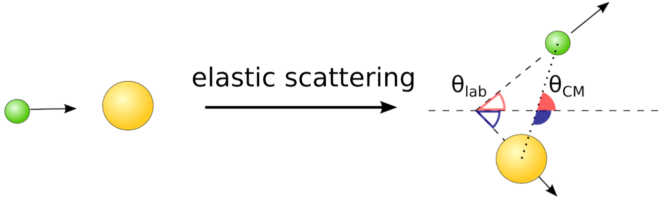
$\frac{d\sigma}{d\Omega}$ - differential cross section - the cross section per unit of a solid angle.

$$\int_0^{\pi}\int_0^{2\pi}\frac{d\sigma}{d\Omega}\sin\theta d\varphi d\theta=\sigma$$



Theoretical Models

Elastic Scattering



Shroedinger's equation:

$$\left[-\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + V(\vec{r}_1, r_2)\right]\psi = E\psi$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

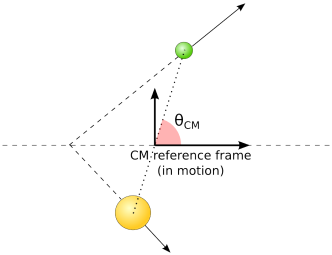
In the CM reference frame:

$$M = m_1 + m_2$$

$$\underbrace{-\frac{\hbar^2}{2M}\nabla_{R_{CM}}^2\psi}_{=\varepsilon_0\psi} \underbrace{-\left(-\frac{\hbar^2}{2\mu}\nabla_r^2 + V(r)\right)\psi}_{=\varepsilon\psi} = E\psi$$

$$\nabla^2 = \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2} + \frac{\partial^2}{\partial z_i^2}$$

Elastic Scattering - Reduction to One-Body Problem



Center of mass motion:

$$-\frac{\hbar^2}{2M}\nabla^2_{R_{CM}}\psi = \varepsilon_0\psi$$

Relative motion:

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2_r + V(r)\right)\psi = \varepsilon\psi$$

Potential of the Interaction

$$V_{\text{eff}} = V_C(r) + V_l(r) + V_N(r) + V_S$$

- Coulomb Potential (R_C is the Coulomb radius)

$$V_C(r) = \frac{Z_1 Z_2 e^2}{r}, \quad r > R_C$$

$$V_C(r) = \frac{Z_1 Z_2 e^2}{2R_C}\left(3 - \frac{r^2}{R_C^2}\right), \quad r < R_C$$

- Centrifugal Potential

$$V_l(r) = \frac{l(l+1)\hbar^2}{2\mu r^2}$$

- Nuclear Potential $\rightarrow$??

Optical Model

Elastic scattering. Imaginary potential $\rightarrow$ absorbtion.

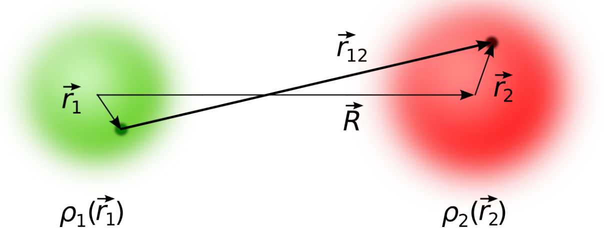
Optical Potential:

$$V_N(r) = V(r) + iW(r)$$

- Phenomenological Potentials
  - Woods-Saxon: $V(r) = \dfrac{V}{1 + e^{(r-R_0)/a}}$
    $$R_0 = r_0(A_{\text{projectile}}^{1/3} + A_{\text{target}}^{1/3})$$
  - Woods-Saxon Squared: $V(|r|) = \dfrac{V}{(1 + e^{(r-R_0)/a})^2}$
- Folding Potential
  $$V(r) = \int\int \rho_1(\vec{r}_1)\ V\ \rho_2(\vec{r}_2)\ \mathrm{d}^3\vec{r}_1 \mathrm{d}^3\vec{r}_1$$

Here we can use different functions for ρ and V

Folding potentials



- Double-folding

$$V_{DF} = \int\int \rho_{proj}(\vec{r}_{proj})\ V(\vec{r}_{12})\ \rho_{targ}(\vec{r}_{targ})\ \mathrm{d}^3\vec{r}_{proj}\mathrm{d}^3\vec{r}_{targ}$$

- Single-folding

$$V_{SF} = \int \rho_{targ}(\vec{r}_{targ})\ V(\vec{r}_{12})\mathrm{d}^3\vec{r}_{targ}$$

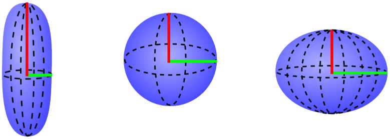
Coupled Channels

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_{opt} + \frac{l(l+1)}{\hbar^2} - E\right]\psi_0 = \begin{cases} 0 \Rightarrow \text{Optical model} \\ U(r)\psi_1 \Rightarrow \text{Inelastic channel} \end{cases}$$

In the Coupled Channels Model, we deform the potential by modifying the radius:

$$R \rightarrow r_0 A^{1/3}[1 + \beta Y_{20}(\theta, \varphi)]$$

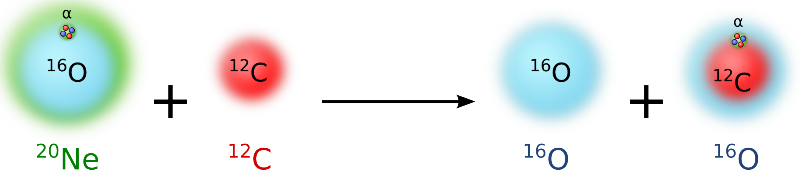
So the potentials are no more spherically symmetrical.



β>0 prolate "cigar" | β=0 symmetric | β<0 oblate "hamburger"

α Cluster Transfer

It is known that Ne and C have α cluster structure $(N\alpha)$

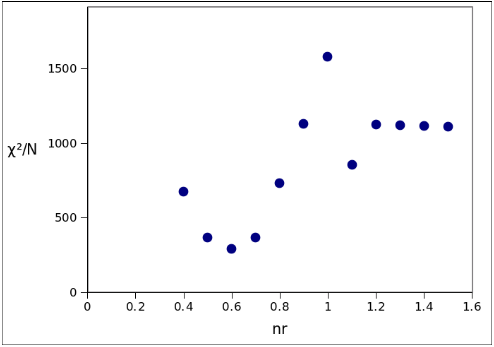


$$V_N = V_{\alpha-\text{O}} + V_{\alpha-\text{C}} + V_{\text{O-C}}$$

Data Analysis with Fresco

Fresco and Searching Fresco (sfresco)

- We used the program `fresco` to do the calculations.
- `sfresco` is the searching version of `fresco`. It tries to optimize the parameters for best fitting.



Analyzing Experimental Data

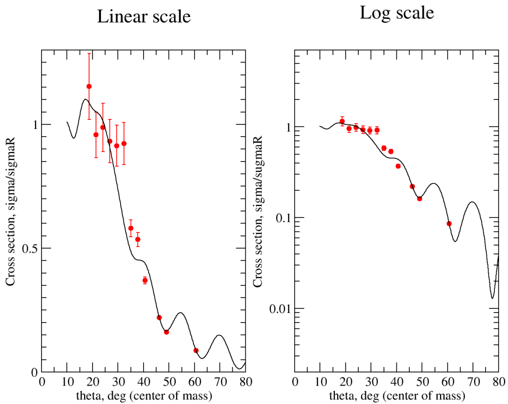
We try to describe the data obtained by Group C

- Optical Model
- Coupled Channels Model
- Alpha Clusters Model

Optical Model - Woods-Saxon Potential

Woods-Saxon Squared for the imaginary part

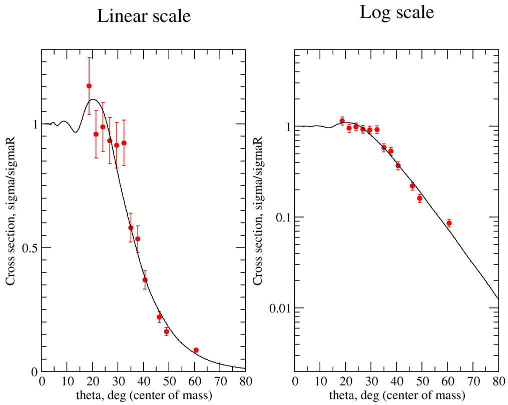
$$V(r) = \frac{V}{1+e^{(r-R_0)/a_0}} \qquad W(r) = \frac{W}{(1+e^{(r-R_i)/a_i})^2}$$



V = 146.2 MeV; r0 = 1.30 fm; a0 = 0.637 fm;
W = 5.77 MeV; ri = 1.28 fm; ai = 0.648 fm

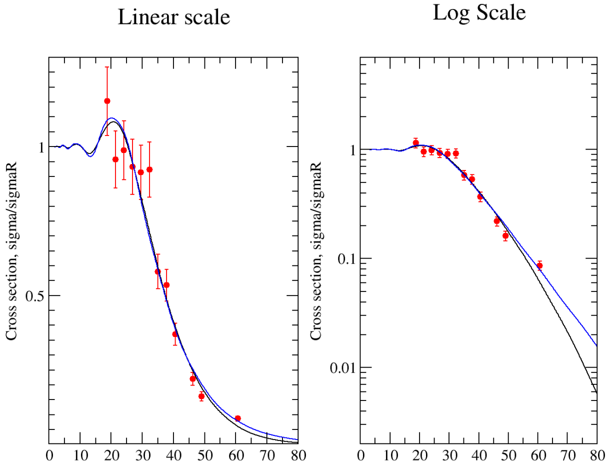
Optical Model - Double-Folding Potential

M3Y double folding potential, Gaussian densities

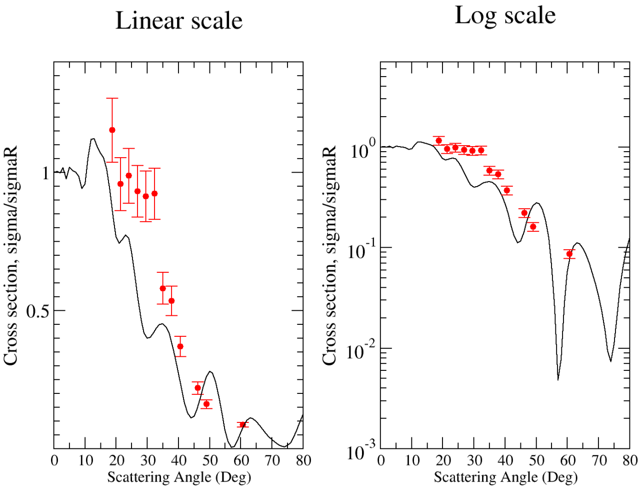


$$N_R = 0.283, \qquad N_i = 0.321$$

Double-Folding Potential: Optical vs. Coupled Channels Models



α Cluster Transfer



Conclusion

- Optical model with pheneomenological and double folding potentials are successful to describe the experimental data.
- We need a deep real potential ($V \approx 150$ MeV) and a shallow imaginary potential ($W \approx 5$ MeV).
- These potential values prove that the interaction of projectile and target nuclei take place at the surface.
- α-transfer should be studied further in the future to understand the following difference in the total cross-sections:

σ_{total}	σ_{CC}	σ_{OM}	$\sigma_{\alpha\text{trans}}$
1142.20 mb	1060.93 mb	1130.82 mb	1583.13 mb

Thanks

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