Low-energy Coulomb excitation

Introduction

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What we are about to do ?

- Few lectures:
  1. Coulex – some basics concerning description of excitation and deexcitation process
  2. What is the GOSIA code – how to start ?
  3. What is the GOSIA gamma-ray yield?
  4. Coulex experiments with stable and RIB beams - the most important aspects.
  5. Coulex as a tool to study shapes of atomic nuclei (Quadrupole Sum Rules method)

What we are about to do ?

- Hands-on sessions:
  1. Starting with GOSIA – declaration of the investigated nucleus (LEVE) and inverse kinematics experiment (EXPT)
  2. How to get the initial set of matrix elements ?
  3. Description of the geometry of the experimental set-up – MINIBALL Ge array + DSSD particle detector
  4. Gamma-ray yields calculations.
  5. Declaration of few experiments with different beam, $\theta_{scattering}$ (normalization).
  6. Minimization and error calculation.

Why Coulex ?

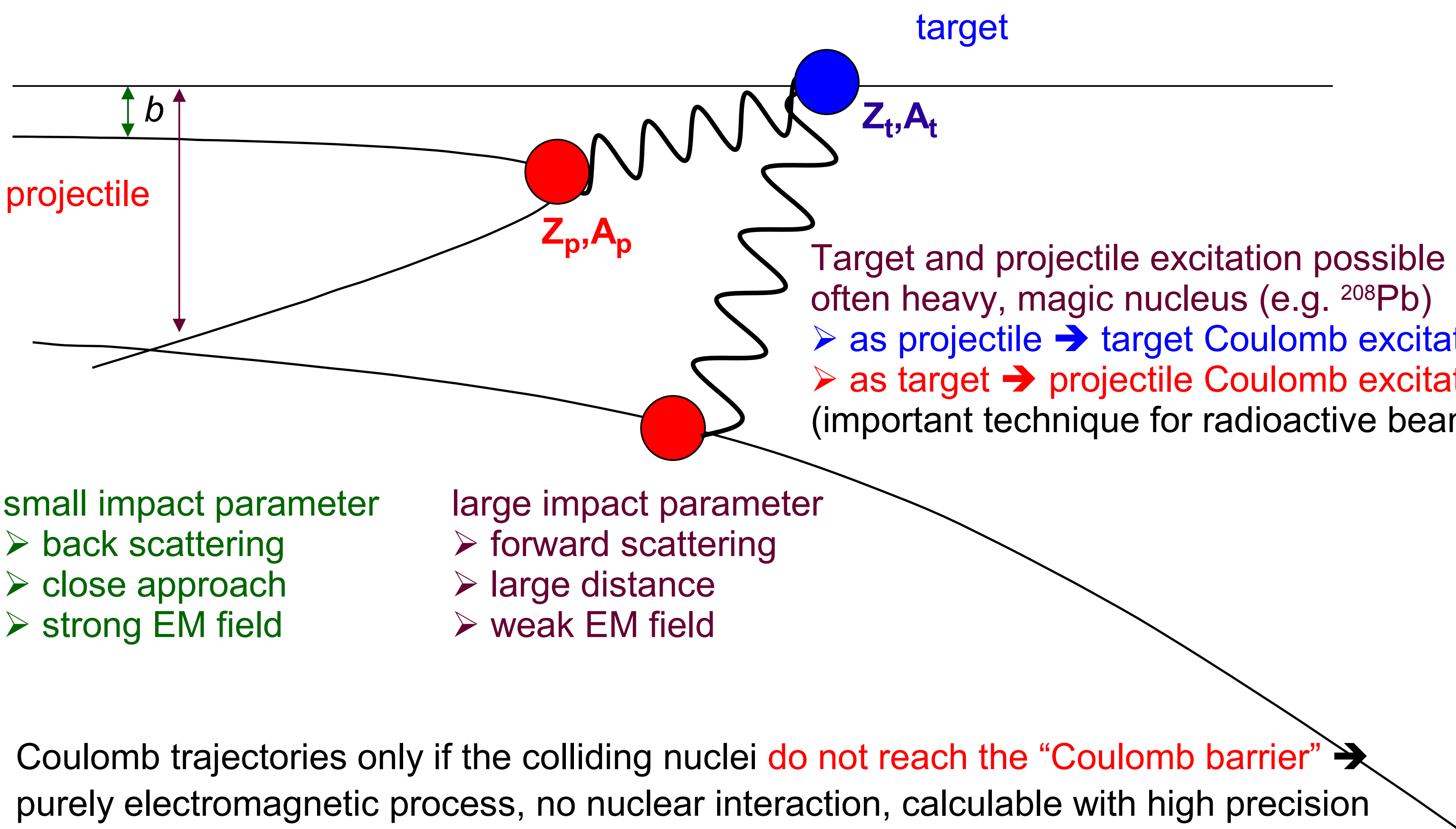
- Response of the nucleus to excitations => nuclear structure => macroscopic shape.
- Shape as a fundamental property of an atomic nucleus.
- **Coulex – the most powerful and direct experimental method to study nuclear collectivity and shapes.**
- Excitation mechanism – purely electromagnetic. The only nuclear properties involved – matrix elements of the electromagnetic multipole moments.
- Nuclear structure studied in a model-independent way
- Bring information on Q_s and relative signs of matrix elements – direct distinguish between prolate and oblate shape

Coulomb excitation – some basics

- Pure electromagnetic interaction if only the distance of closest approach D_{min} is at least 5 fm - nuclear part of the interaction can be neglected (Cline's criterion)

$$D_{min} \geq r_s = [1.25\,(A_1^{1/3} + A_2^{1/3}) + 5]\ \text{fm}$$

- The excitation process depends on: $\mathbf{E_{beam}}$, **Z** of projectile and target nuclei, $\boldsymbol{\theta_{scattering}}$



Target and projectile excitation possible
often heavy, magic nucleus (e.g. ^{208}Pb)
- as projectile ➔ target Coulomb excitation
- as target ➔ projectile Coulomb excitation

(important technique for radioactive beams)

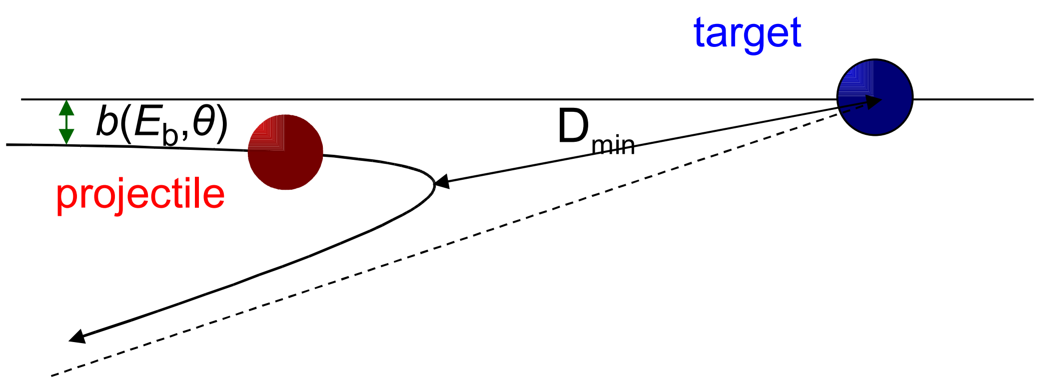
small impact parameter
- back scattering
- close approach
- strong EM field

large impact parameter
- forward scattering
- large distance
- weak EM field

Coulomb trajectories only if the colliding nuclei do not reach the “Coulomb barrier” ➔ purely electromagnetic process, no nuclear interaction, calculable with high precision

„Safe“ bombarding energy requirement

is a consequence of the D_{min} requirement



$$E_b(\theta_{cm}) = 0.72 \cdot \frac{Z_P Z_T}{D_{min}} \cdot \frac{A_p + A_t}{A_t} \cdot \left[1 + \frac{1}{\sin\left(\frac{\theta_{cm}}{2}\right)}\right] [\mathrm{MeV}]$$

Preparing the experiment using the:

➔ choose adequate beam energy ($D > D_{min}$ for all θ)
low-energy Coulomb excitation

➔ limit scattering angle, i.e. select impact parameter $b\ (E_b,\ \theta) > D_{min}$
high-energy Coulomb excitation

Electromagnetic interaction well-known → for a given set of matrix elements the Coulomb excitation cross section for any states of the investigated nucleus may be calculated

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Straight-forward method: quantum mechanical treatment → expanding the total wave function into eigenstates of the relative orbital angular → high number of partial waves, quantal coupled channel equations...

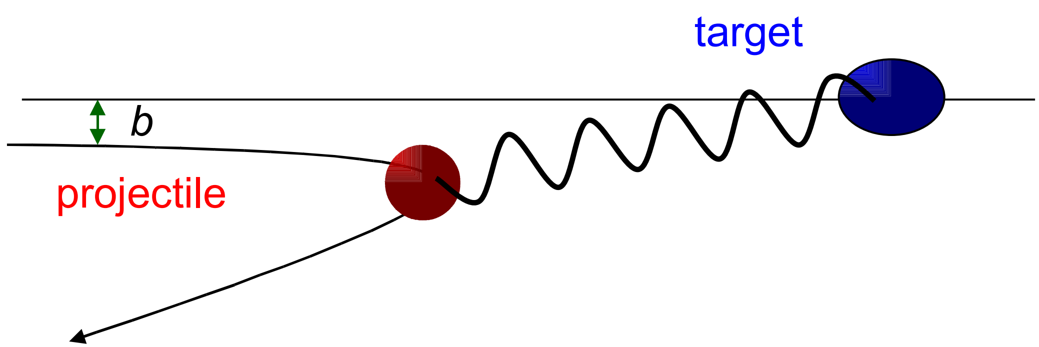
IMPRACTICAL !!

↓

Simplified and replaced by a semiclassical approach without any significant loss of accuracy

Semiclassical picture of the Coulomb excitation

- Projectile is moving along the hyperbolic orbit and the nuclear excitation is caused by the time-dependent electromagnetic field from the projectile acting on the target nucleus
- **Assumption:** trajectories can be described by the classical equations of motion, electromagnetic interaction is described using the quantum mechanic.



- Validity of semiclassical approach:

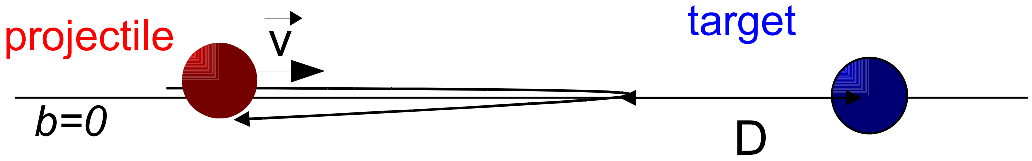
  1. $\lambda_{projectile} << D_{min}$ for a head on collision,

  2. small energy transfer,

  3. the excitation is induced only by the monopole-multipole interaction,

  4. time seperation of the collision ($10^{-19} – 10^{-20}$ s) and deexcitation (10^{-12} s) process.

Ad 1. Validity of classical Coulomb trajectories

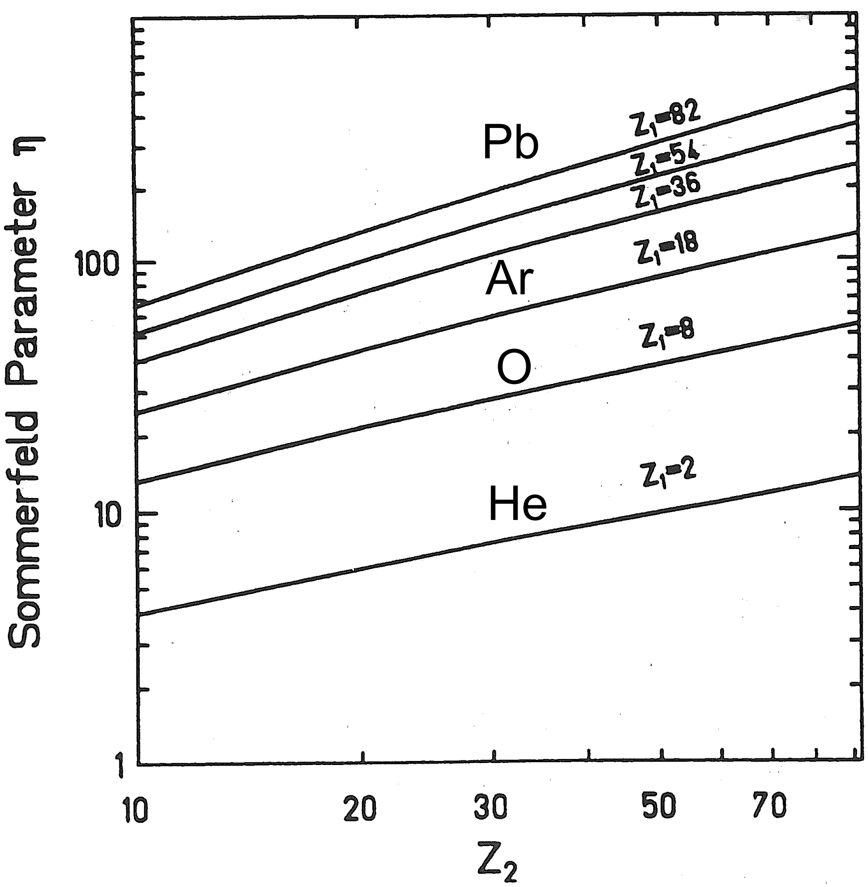


$\lambda_{projectile}$ << D => Sommerfeld parameter η

$$\eta = \frac{D}{2\lambda\!\!\!^{-}} = \frac{Z_P Z_T e^2}{\hbar v} >> 1$$

η >> 1 requirement for a semiclassical treatment of equations of motion → hyperbolic trajectories

- ➢ condition very good fulfilled in heavy ion induced Coulomb excitation
- ➢ equivalent to the number of exchanged photons needed to force the nuclei on a hyperbolic orbit



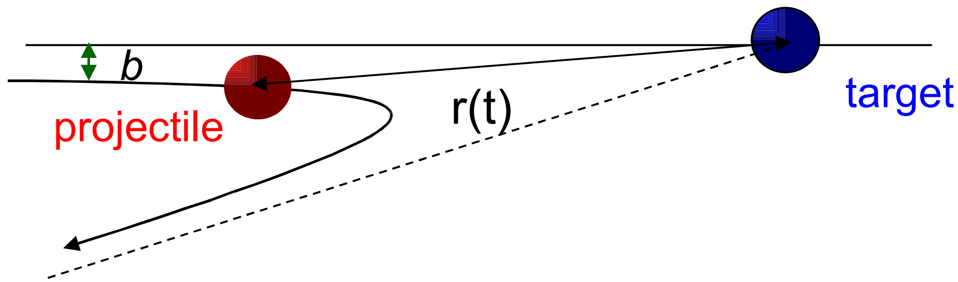
Semiclassical treatment is expected to deviate from the exact calculation by terms of the order ~ 1/η

Ad 2. Validity of semiclassical approach
small energy transfer

- Modification of the trajectory due to the energy transfer
- In the classical kinematics picture the point of the energy transfer is not known $\rightarrow$ accurate determination of energy transfer effects is not possible
- To the 1st order the energy transfer effect can be described by the symmetrization of relevant excitation parameters – average of perturbed and unperturbed orbits parameters.
- Symmetrization procedure is adequate when $E_{exc} << E_{beam}$

~ 1-3 MeV   ~ 100 MeV

Coulomb excitation theory - the general approach



The excitation process can be described by the time-dependent *H*:

$$H = H_P + H_T + V\,(r(t))$$

with $H_{P/T}$ being the free Hamiltonian of the projectile/target nucleus
and $V(t)$ being the time-dependent electromagnetic interaction
(remark: often only target or projectile excitation are treated)

Denoting the P/T wave function by $\psi(t)$ the time-dependent Schrödinger equation:

$$i\hbar\, d\psi(t)/dt = [H_P + H_T + V\,(r(t))]\;\psi(t)$$

During the collision, the wave function can be expressed as time-dependent expansion $\psi(t) = \sum_n a_n(t)\,\phi_n$ of the eigenstates ϕ_n of free $H_{P/T}$ what leads to a set of coupled equations for the **time-dependent excitation amplitudes $a_n(t)$**

$$i\hbar\, da_n(t)/dt = \sum_m \langle\phi_n|V(t)|\,\phi_m\rangle \exp[i/\hbar\,(E_n-E_m)\,t]\;a_m(t)$$

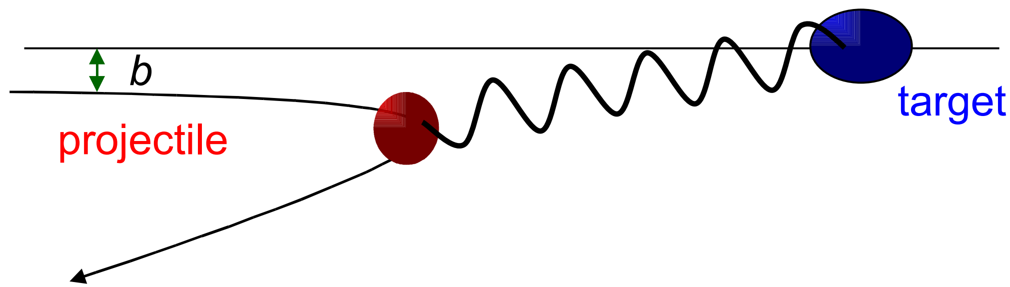
m - all states involved in the excitation process
→ nr. of coupled equations

can be written as an expansion of multipoles

Energies of initial and final states

Coulomb excitation theory - the general approach

The coupled equations for $a_n(t)$ are usually solved by a multipole expansion of the electromagnetic interaction V(r(t))



$V_{P\text{-}T}(r) = Z_T Z_P e^2/r$ — monopole-monopole (Rutherford) term

$+ \sum_{\lambda\mu} V_P(E\lambda,\mu)$ — electric multipole-monopole target excitation,

$+ \sum_{\lambda\mu} V_T(E\lambda,\mu)$ — electric multipole-monopole project. excitation,

$+ \sum_{\lambda\mu} V_P(M\lambda,\mu)$
$+ \sum_{\lambda\mu} V_T(M\lambda,\mu)$ — magnetic multipole project./target excitation (but small at low v/c)

$+ O(\sigma\lambda,\sigma'\lambda'>0)$ — higher order multipole-multipole terms (small)

Coupled equations

$$i\hbar\, da_n(t)/dt = \sum_m \langle \phi_n | V(t, T\lambda, \mu) | \phi_m \rangle \exp[i/\hbar\,(E_n - E_m)\,t]\, a_m(t)$$

In the heavy ion induced Coulomb excitation the interaction strength gives rise to multiple Coulomb excitation

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nuclear state can be populated indirectly, via several intermediate states

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The exact excitation pattern is not known
The excitation probability of a given excited state might strongly dependent on many different matrix elements.

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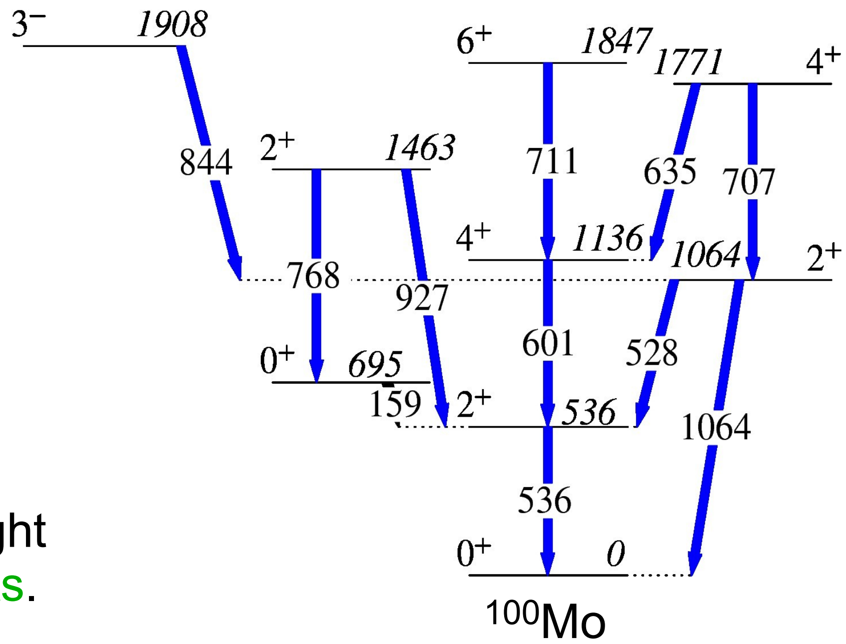


Coulex, HIL, Warsaw, 2007

High number of coupled equations for the $da_n(t)/dt$ -> GOSIA code

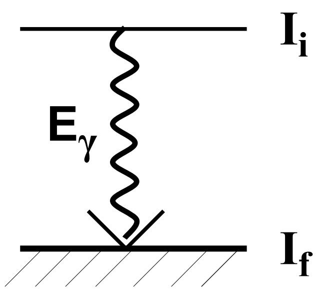
Deexcitation process

- For a given set of matrix elements (Tλ,μ) GOSIA solves differential coupled equations for the **time-dependent excitation amplitudes $a_n(t)$**

  $i\hbar \, da_n(t)/dt = \sum_m \langle \phi_n | \sum_{\lambda,\mu} V(t, T\lambda,\mu) | \phi_m \rangle \exp[i/\hbar \, (E_n - E_m) \, t] \, a_m(t)$

  to find level populations and gamma yields.

- The same set of **Tλ,μ** describes the deexcitation process



$$P(T\lambda;\, I_i \to I_f) = \frac{8\pi(\lambda+1)}{\lambda((2\lambda+1)!!)^2} \cdot \frac{1}{\hbar} \cdot \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} \cdot B(T\lambda;\, I_i \to I_f)$$

$$B(T\lambda;\, I_i \to I_f) = \frac{1}{2I_i+1} \cdot \langle I_f | M(T\lambda(| I_i \rangle^2$$

Calculation includes effects influencing γ-ray intensities: internal conversion, size of Ge, γ-ray angular distribution, deorientation

Summary

- Coulomb excitation is a purely electro-magnetic excitation process of nuclear states due to the Coulomb field of two colliding nuclei.
- The only nuclear properties involved – matrix elements.
- Coulomb excitation is a very precise tool to measure the collectivity of nuclear excitations and in particular nuclear shapes.
- Pure electro-magnetic interaction (which can be readily calculated without the knowledge of optical potentials etc.) requires “safe” distance between the partners at all times.