Beta Delayed Particle Emission

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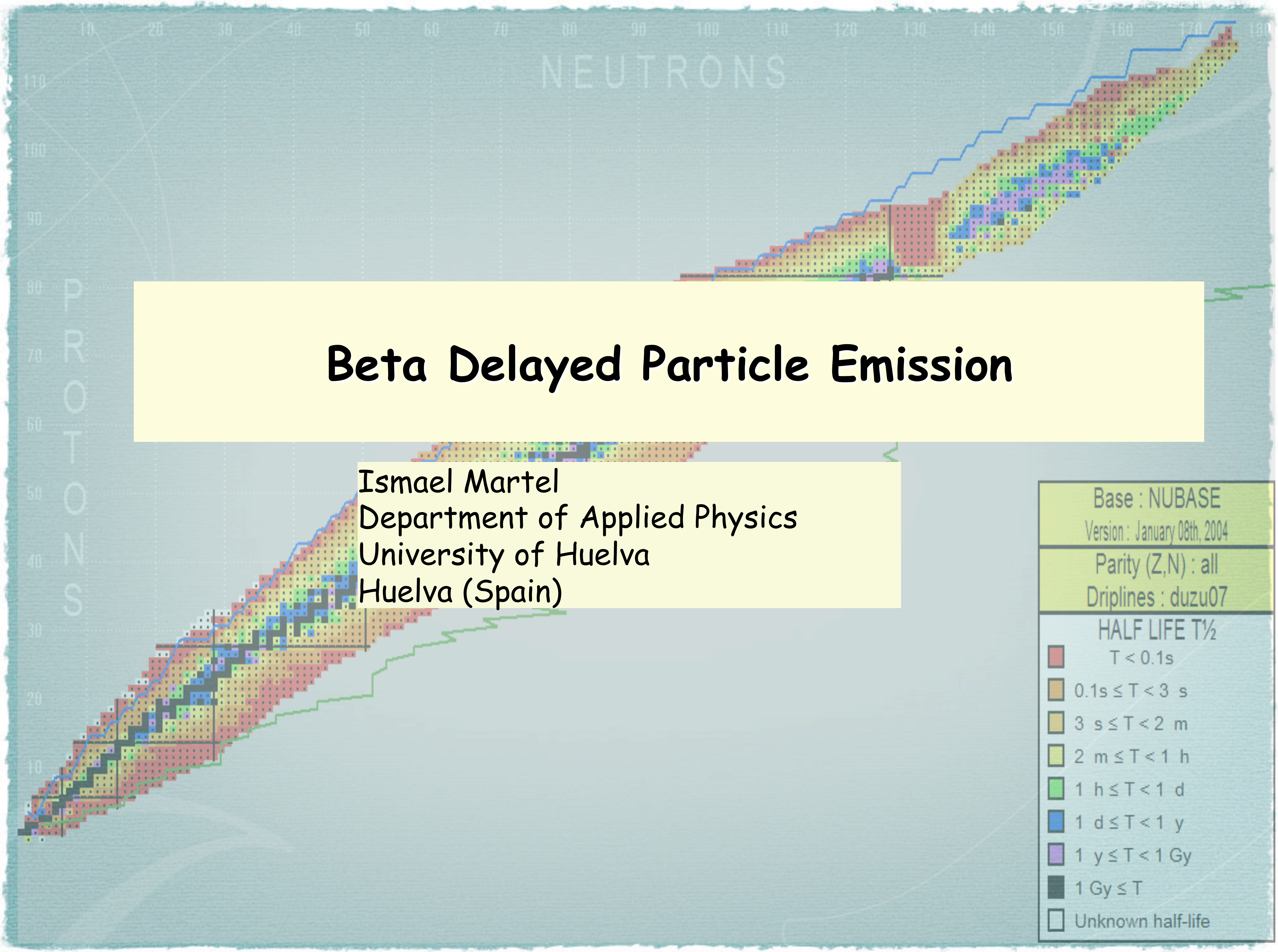
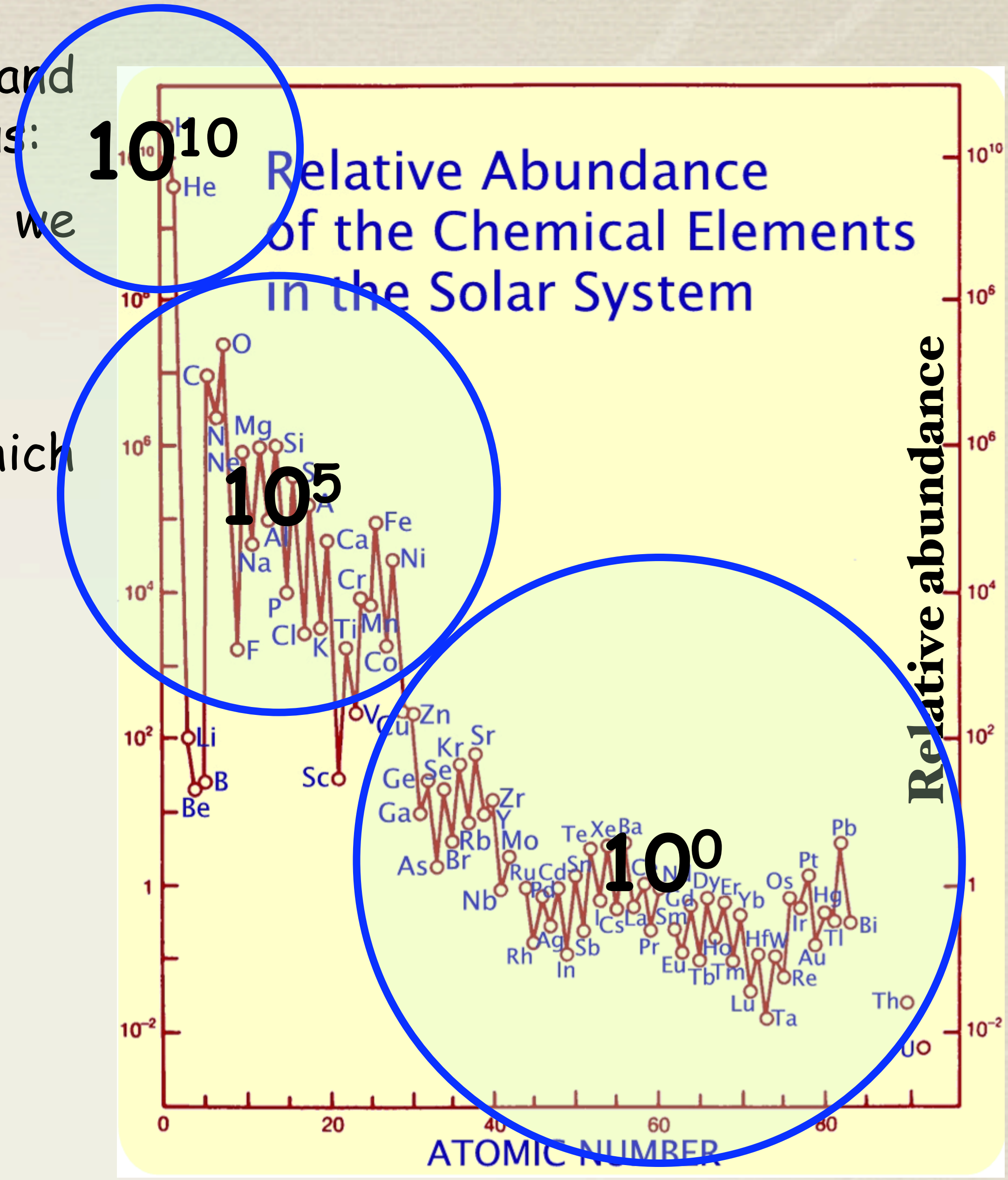


Introduction

Along history, there has been a constant effort to understand the structure and mechanism of the nature that surround us:

- Why the Universe and the Nature have the structure we observe?
- Which are the basic constituents of matter?
- How the different building blocks of matter interact which each other?
- Where, when and how the Universe has been originated?



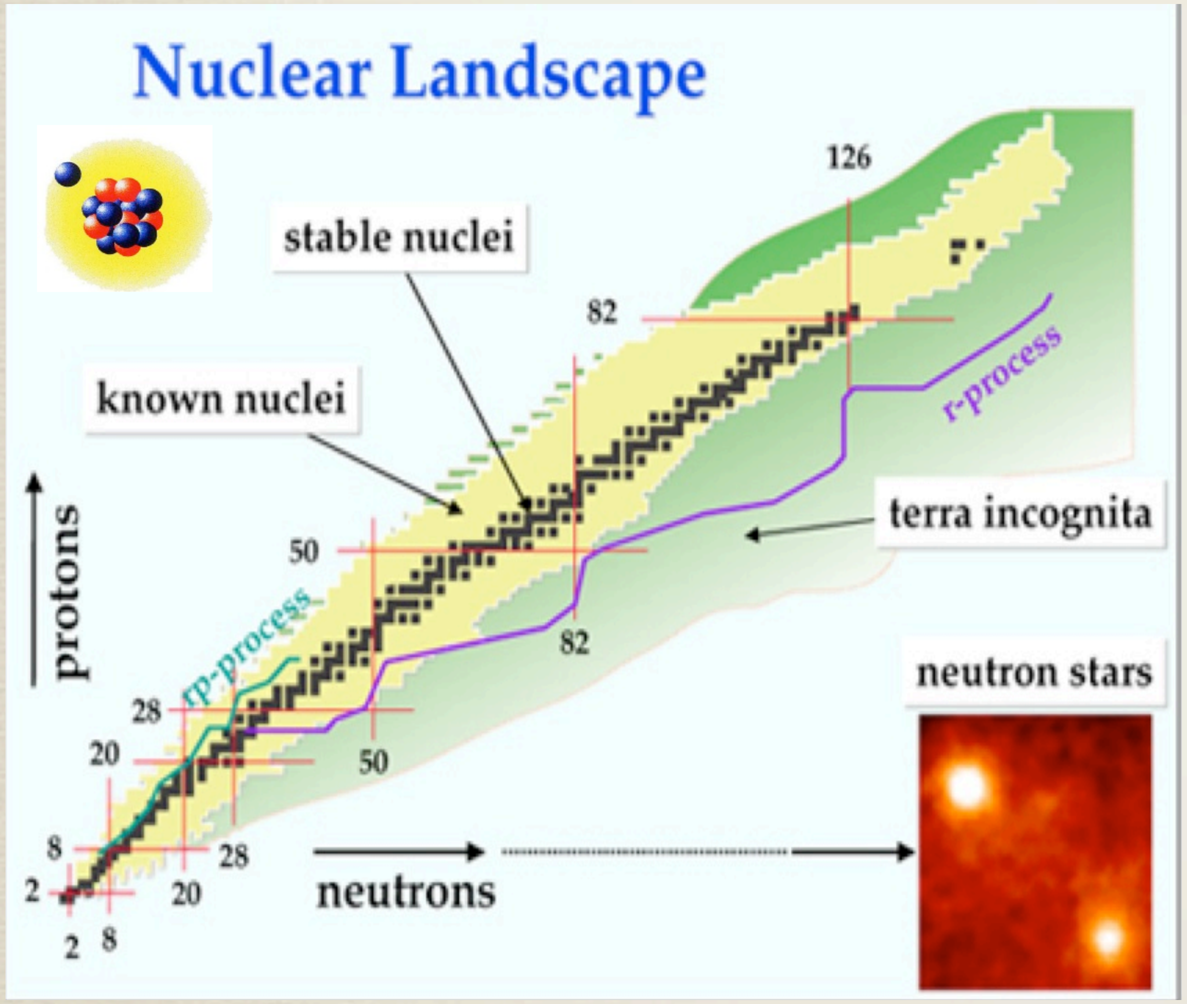
"Creation pillars", nucleosynthesis of stars at Eagle's Nebulae

The research efforts carried out in basic nuclear physics (and Science in a wide sense) along last century (XX) has provided an un-precedent knowledge of the subatomic structure of matter and its constituents, its dynamics and the Origin of the Universe itself.

From a historical point of view, the major steps in the understanding of the Universe have taken place in particle accelerators.

At present Radioctive Beam Facilities we can customize our nuclear system (N,Z), "fabricate" any nucleus controlling the number of constituent protons and neutrons.

Proton Rich Nuclei ← → Neutron Rich Nuclei← → Light unbound systems ← → Super-heavy's



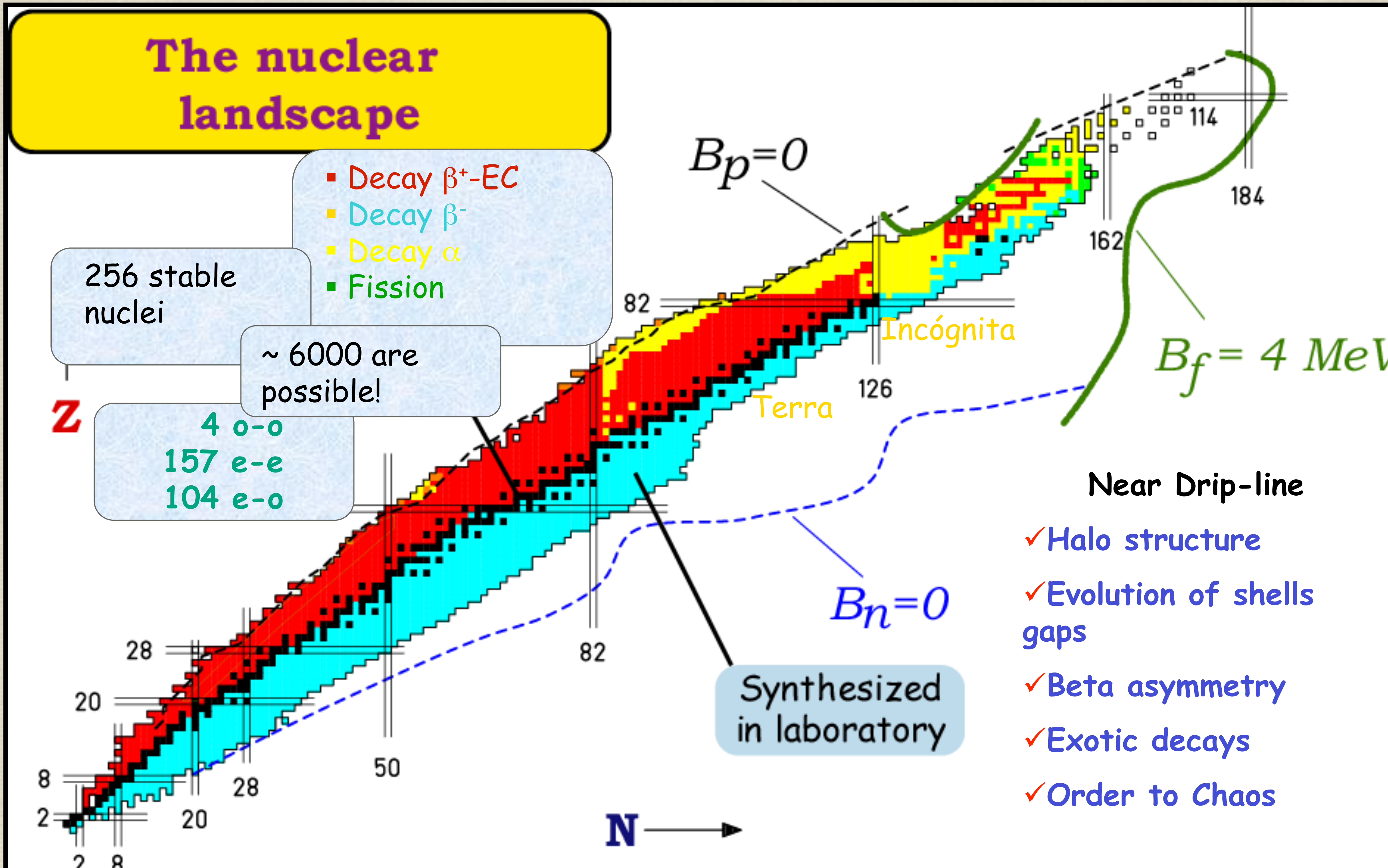
→ Evolution of nuclear structure and nuclear dynamics,

→ Exotic (N,Z) combinations → isospin degree of freedom
- Evolution of shell structure, phase shape transitions, nucleon-nucleon pairing, spin-orbit interaction
- Halo, skin, cluster nuclear structures
- Beyond the drip lines → unbound nuclei & resonances
- Exotic decay modes and Reaction dynamics of exotic systems
- Test of astrophysical scenarios → nuclear astrophysics

Spectroscopic tools → Particle Detectors + Accelerators

Theoretical tools: Precise knowledge of theoretical framework **well tested** with stable nuclei

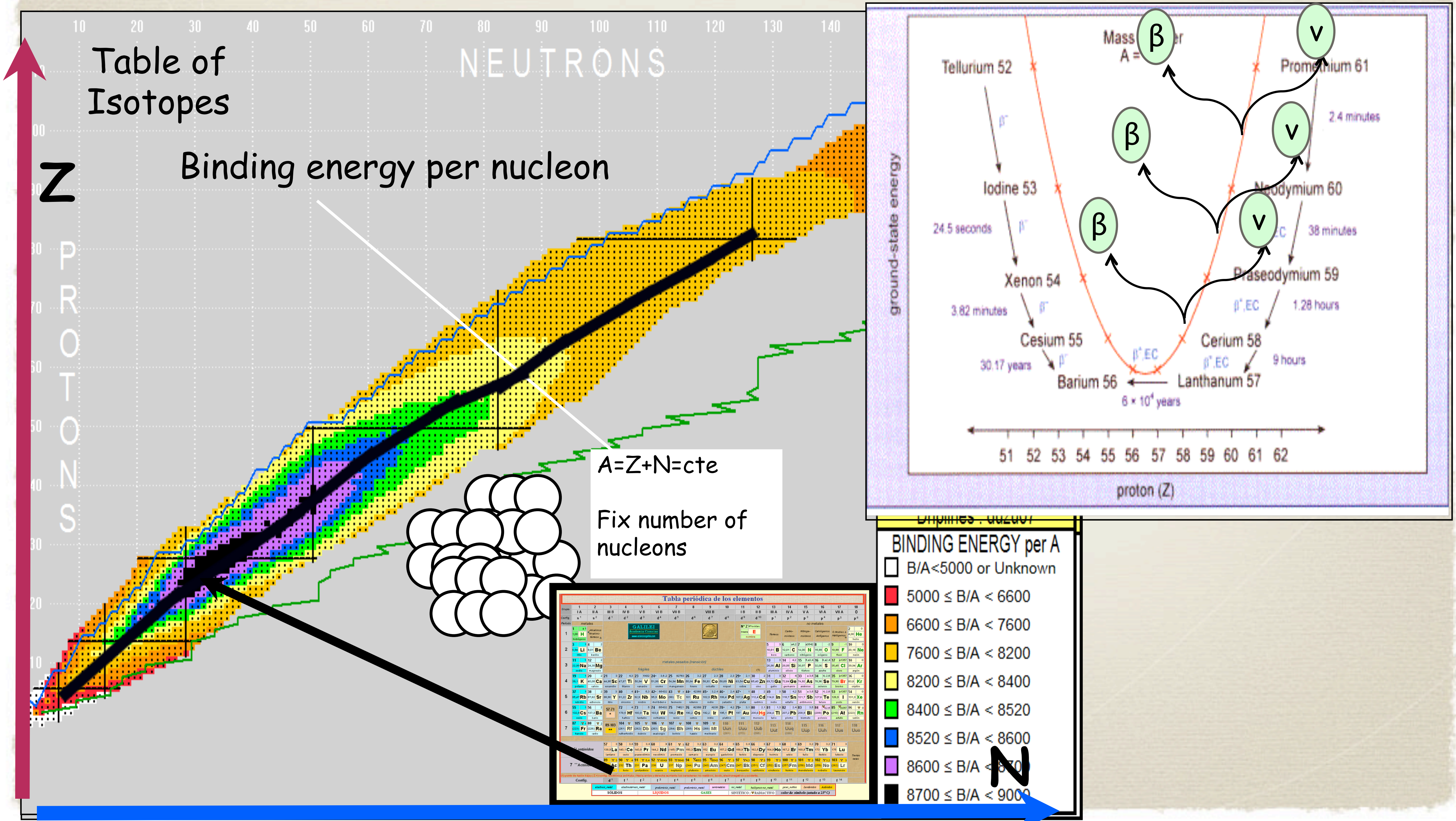
The nuclear landscape

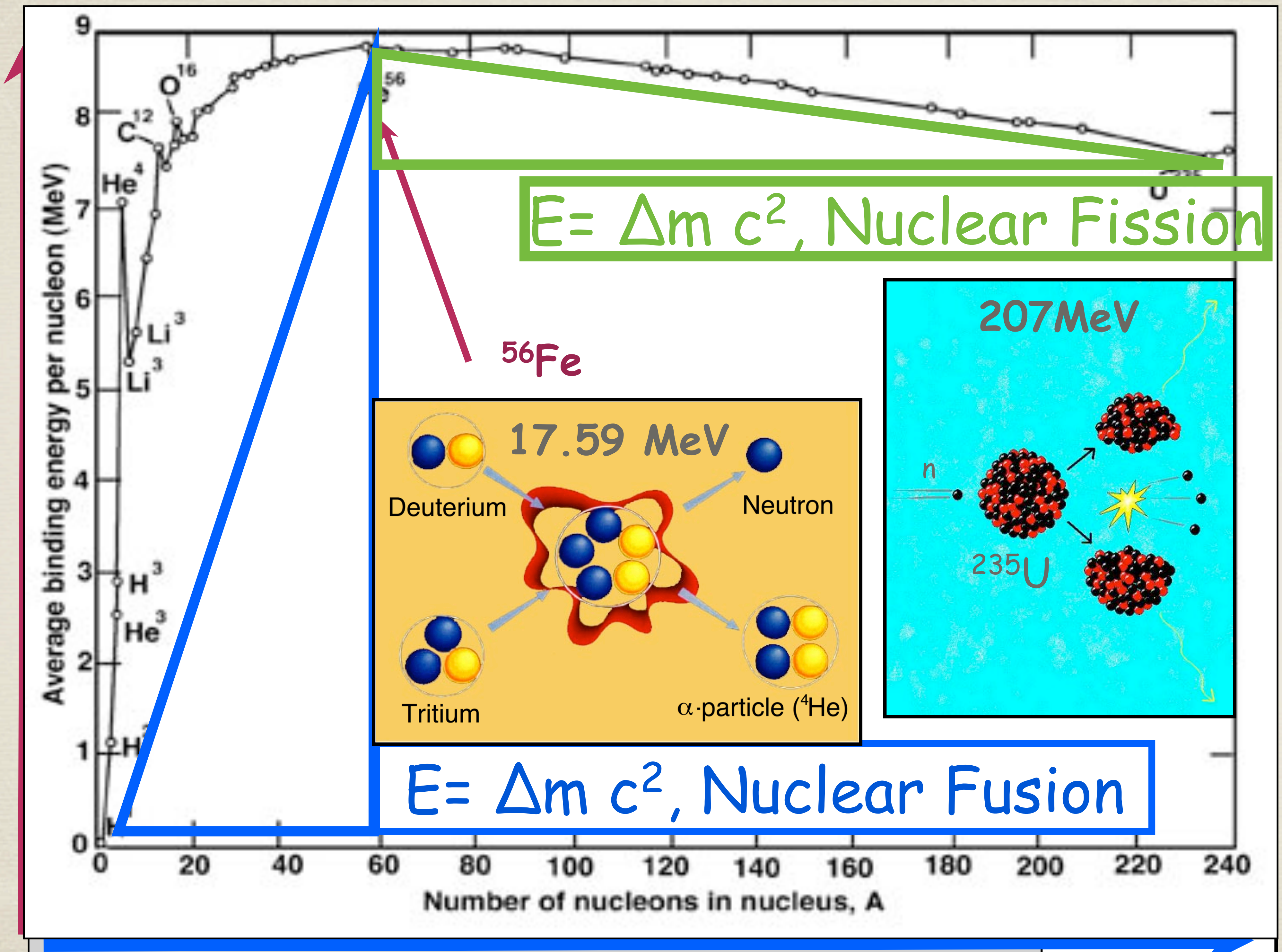


Nuclear stability and radioactivity

Atomic nuclei are very "particular" systems → only "magic" combinations of (Z,N) are possible → stable nuclei
→ nuclear interaction/ nucl. Structure

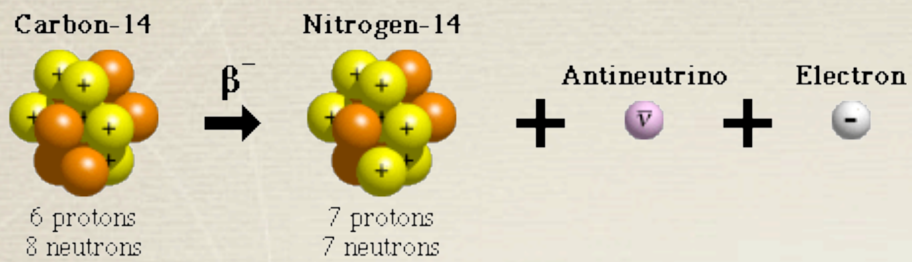
Far from "stable" configurations → excess of energy →
nucleons tends to reorganize and release it
→ weak nuclear force, strong nuclear force, Coulomb force → radioactive decay or **radioactivity**.




Average binding energy per nucleon (MeV)
Number of nucleons in nucleus, A
E= Δm c², Nuclear Fission
E= Δm c², Nuclear Fusion
56Fe
17.59 MeV
Deuterium
Neutron
Tritium
α·particle (4He)
207MeV
n
235U

Some common types of radioactivity

Beta -



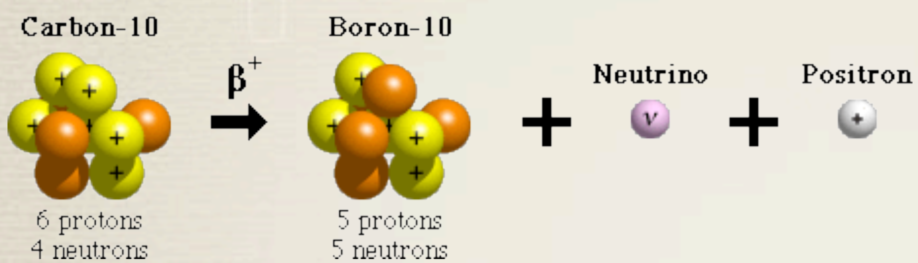
A neutron is transformed in a proton, with the emission of one electron and one antineutrino;

Z => Z+1

N => N-1

A= constant

Beta +

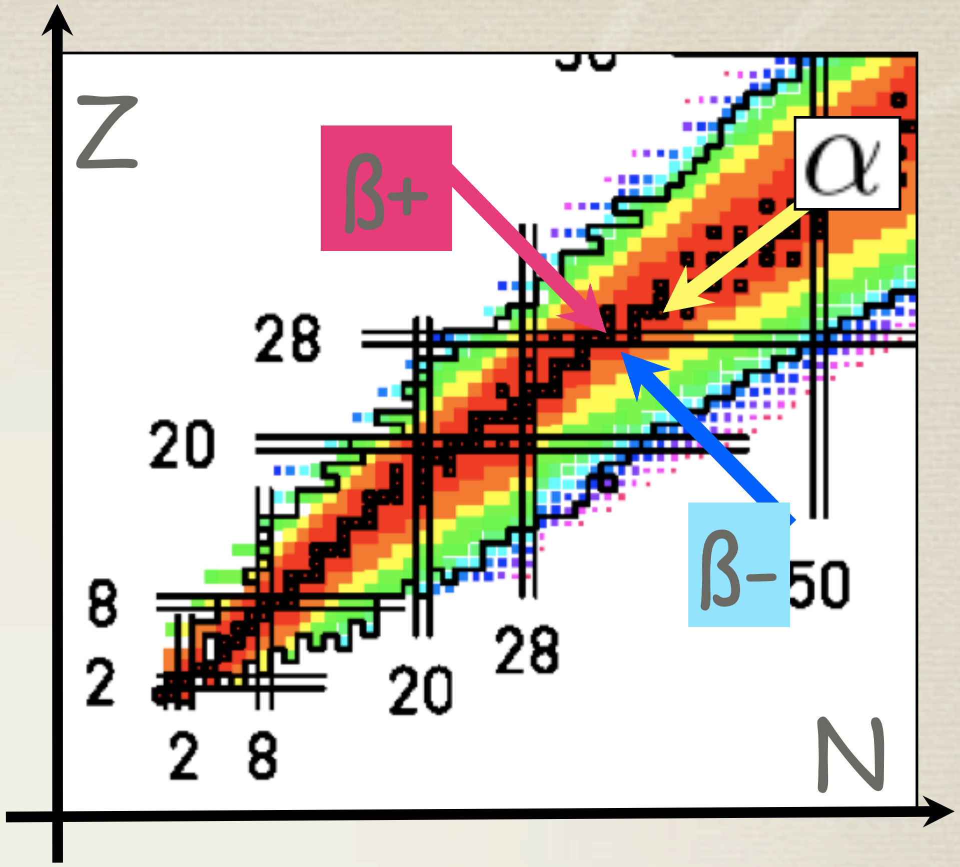


A proton is transformed in a neutron, with the emission of a positron and one neutrino:

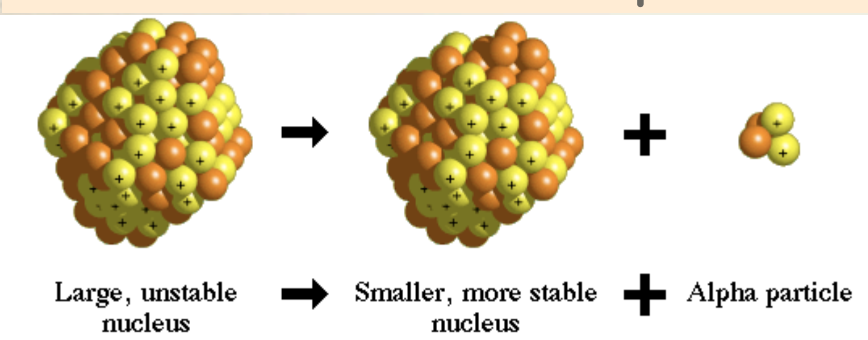
Z => Z - 1

N => N+1

A= constant



Particle emission: Alpha



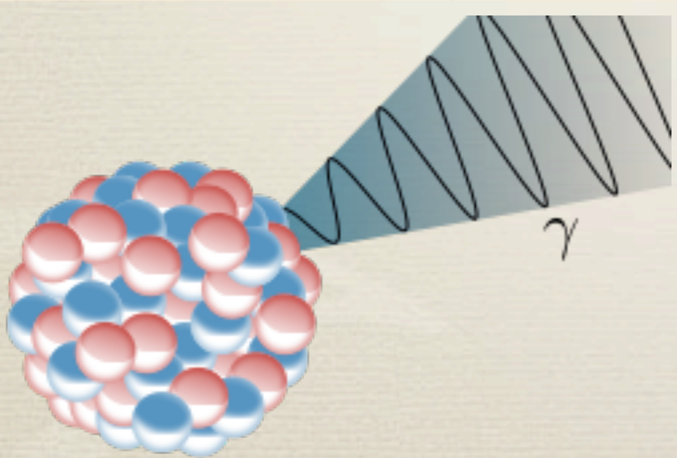
High mass nuclei can decay by emission of a helium nucleus;

Z => Z-2

N => N - 2

A=> A-4

Gamma decay



Emission of electromagnetic radiation (photons) occurs during transitions between nuclear states of higher to lower energies.

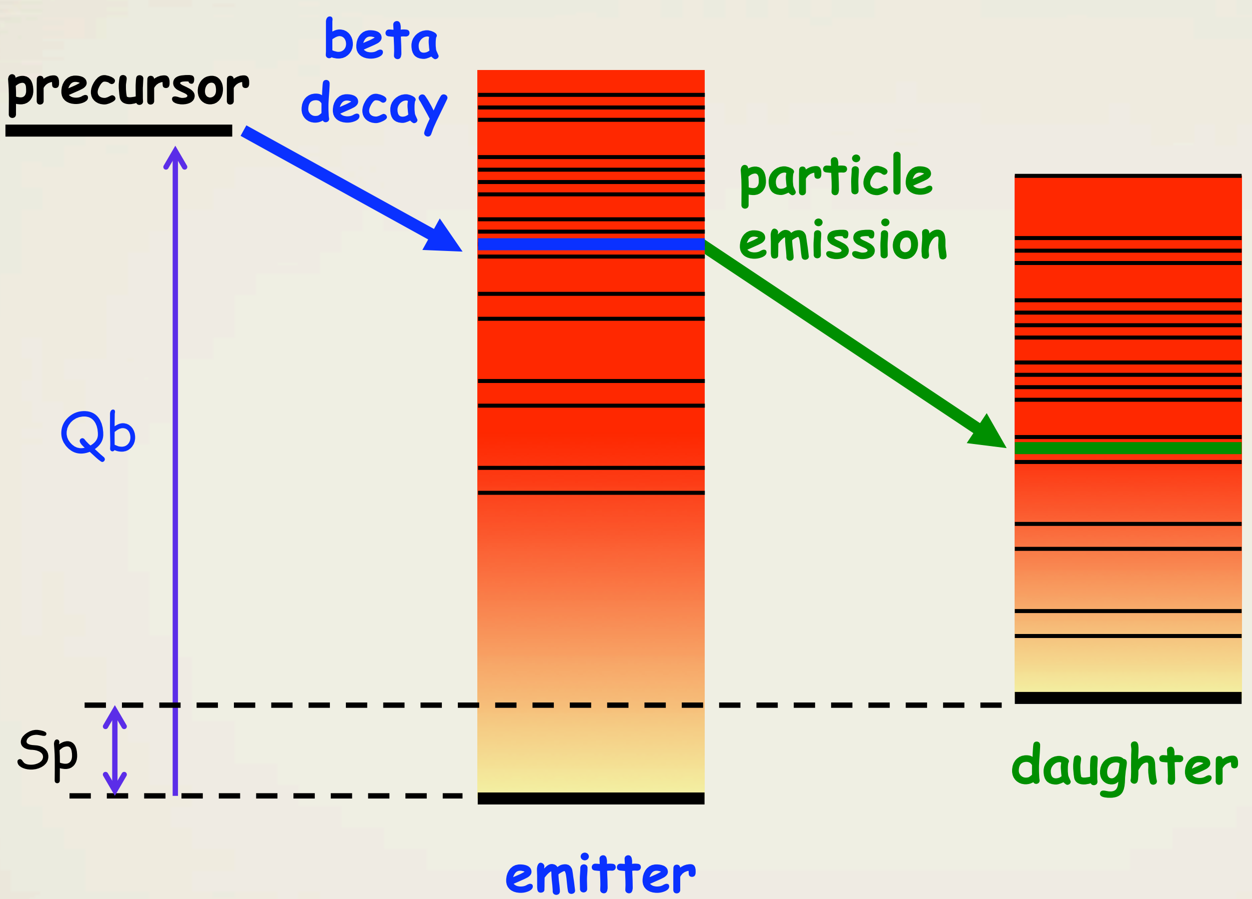
→ NO change in (N,Z) or A values

Beta delayed particle emission

Emission of particles from nuclear (excited) states populated by the beta decay

Two processes:

- Beta decay from the parent nucleus (precursor)
- Particle emission from excited states of "emitter" nucleus



E, Γ

Level density

Spin, Isospin

β-decay properties

→ beta decay to excited levels of "emitter" nucleus; if the excited state is over separation energy Sp → emission of particles

→ The half-life of beta decay is much longer than the nuclear level of emitter, the half-life of the process is given by the beta decay → "***beta - delayed ….***"

history → observed since early stages of nuclear physics:

Beta delayed alphas ($\beta\alpha$): Rutherford (1916) [*Philos. Mag.* ***31*** *(1916) 379*]
→ "Long range alpha particles followed by beta decay of 212Bi"

Beta delayed protons (**βp):** Marsden (1914) → $^{14}N(\alpha,p)^{17}O$ [*Philos. Mag.* ***37*** *(1919) 537*]; Álvarez (1950) bombarded ^{10}B and ^{20}Ne with 32 MeV protons → beta delayed ^{8}B, ^{20}Na α-emitters

The modern era begins in 1960's (βp, β2p)/ Zeldovich, Karnaukhov, Goldansky…
[*Goldanskii NPA 19 (1960) 482*]

→ Information about level energies, spins and parities of participant nuclei (precursor, emitter, daughter)

→ Fundamental physics (Standard Model)

At present days investigations on beta delayed radioactivity are very intense, particularly with the use of radioactive beams:
typical decay mechanism at drip lines

- Large Qb values → access high energy states

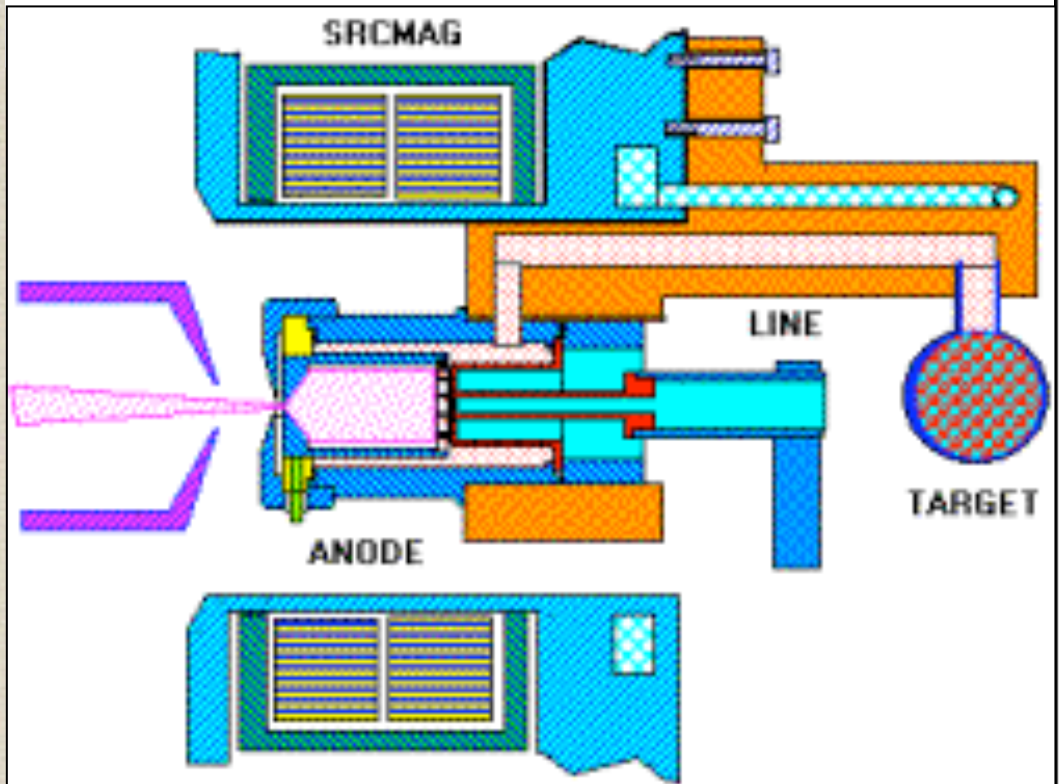
Good alternative to gamma spectroscopy and nuclear reactions limited by beam intensities ~10^4 pps

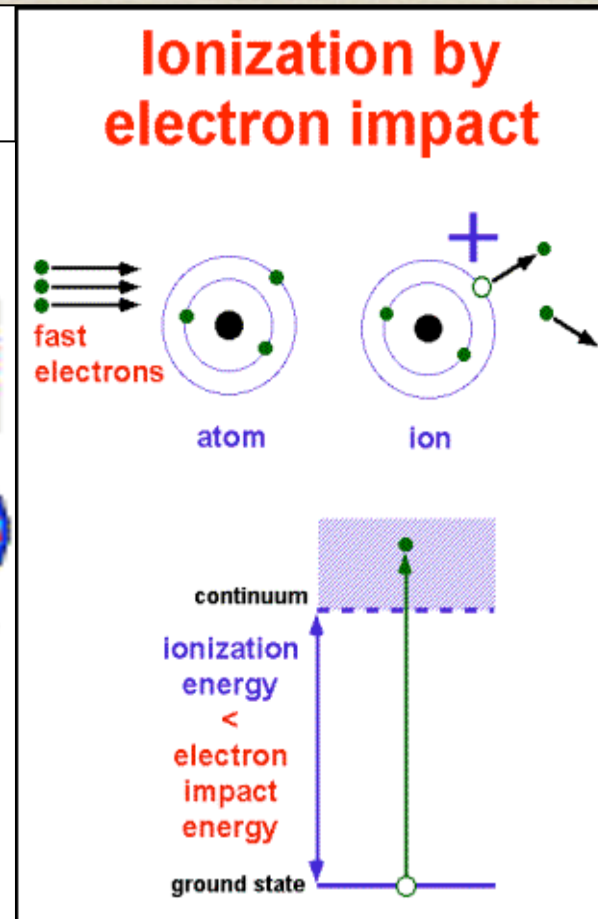
- Beta delayed particle emission → limited by selection rules of beta decay

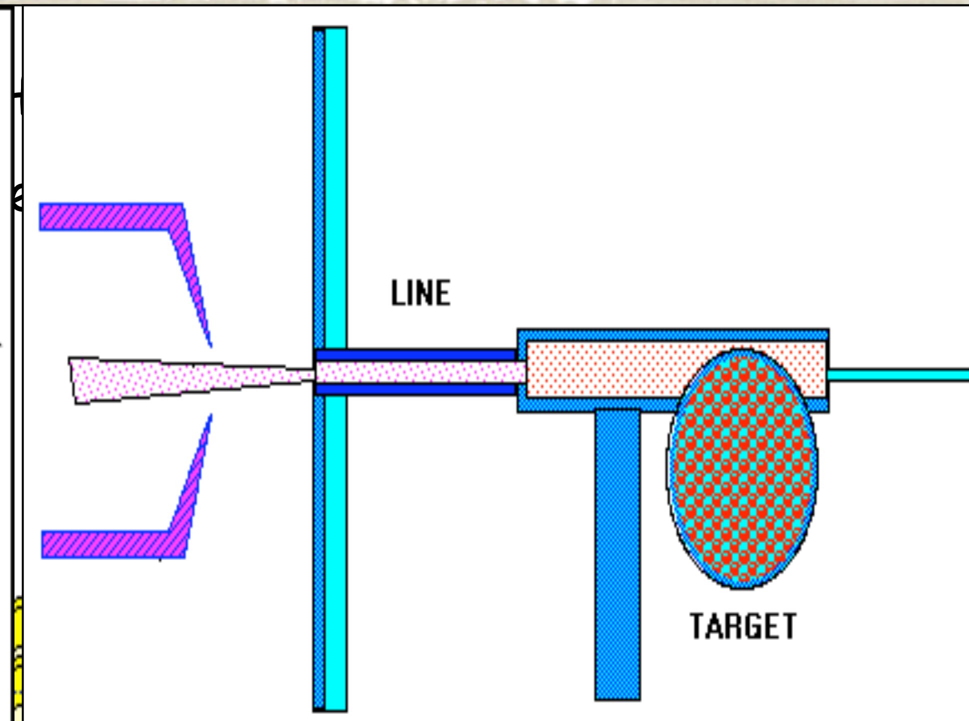
- Usually first type of studies close to drip lines → low isotope production → largest yields obtained directly after ion source and implanted on decay foil.

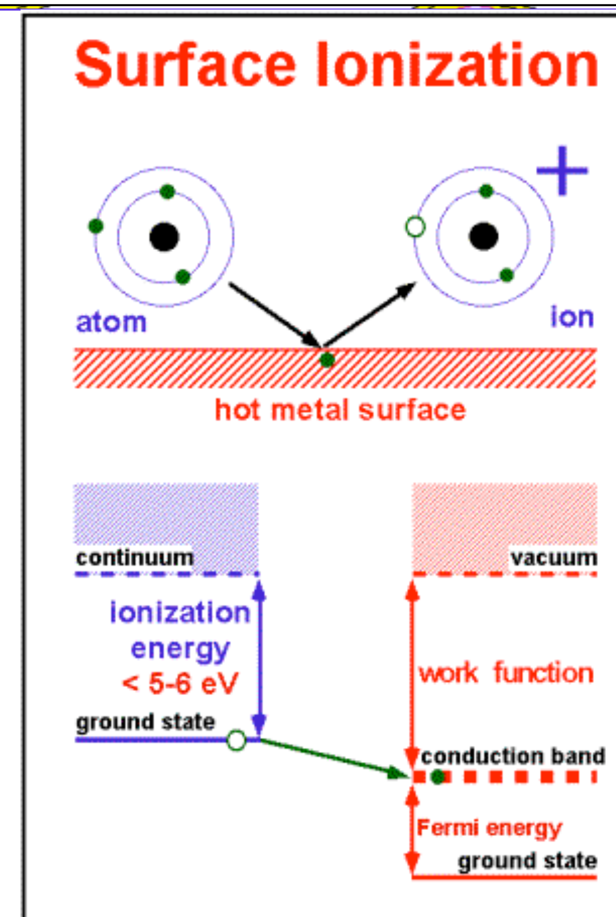
- Relatively "simple" experimental setups.

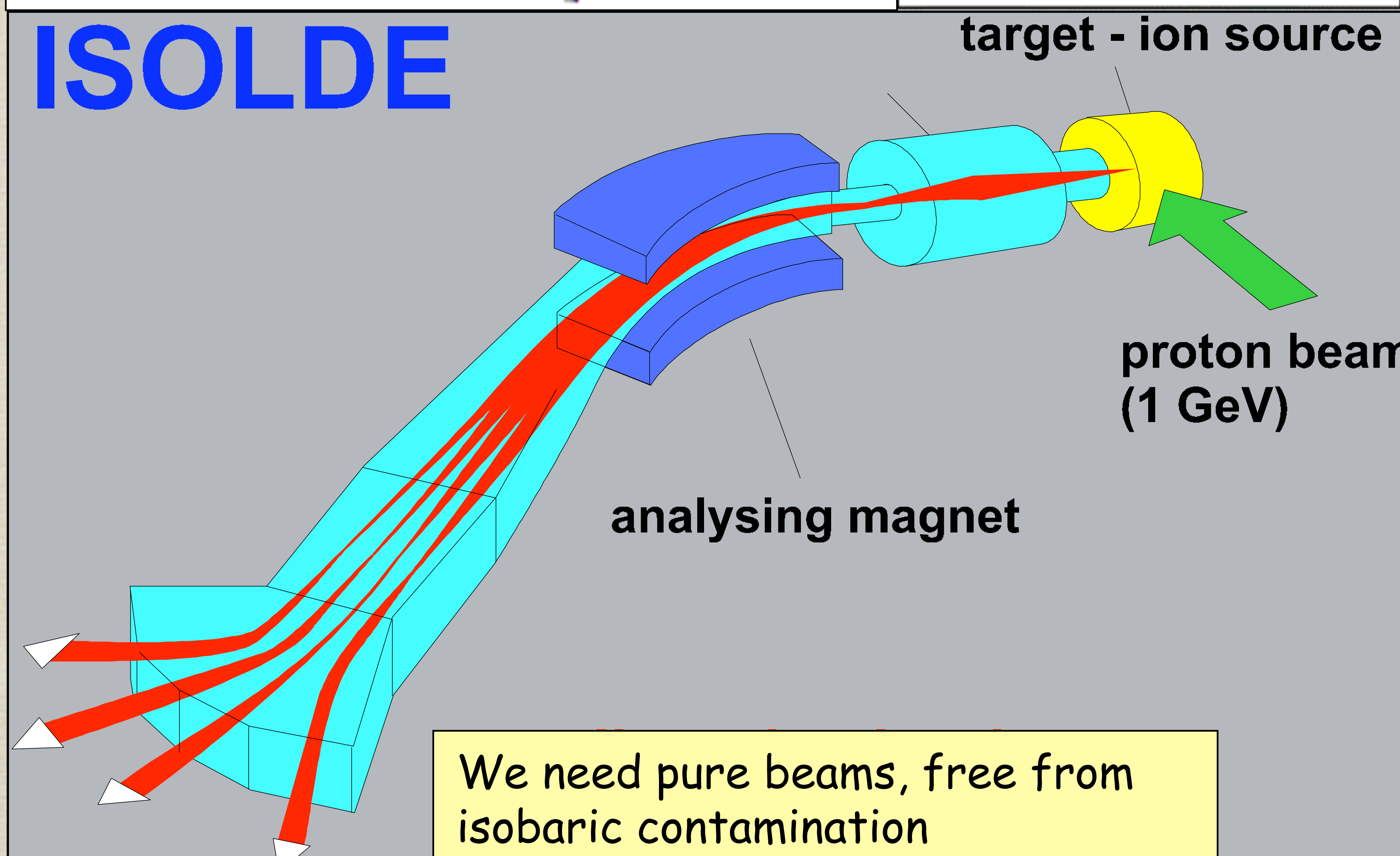
Plasma ion source







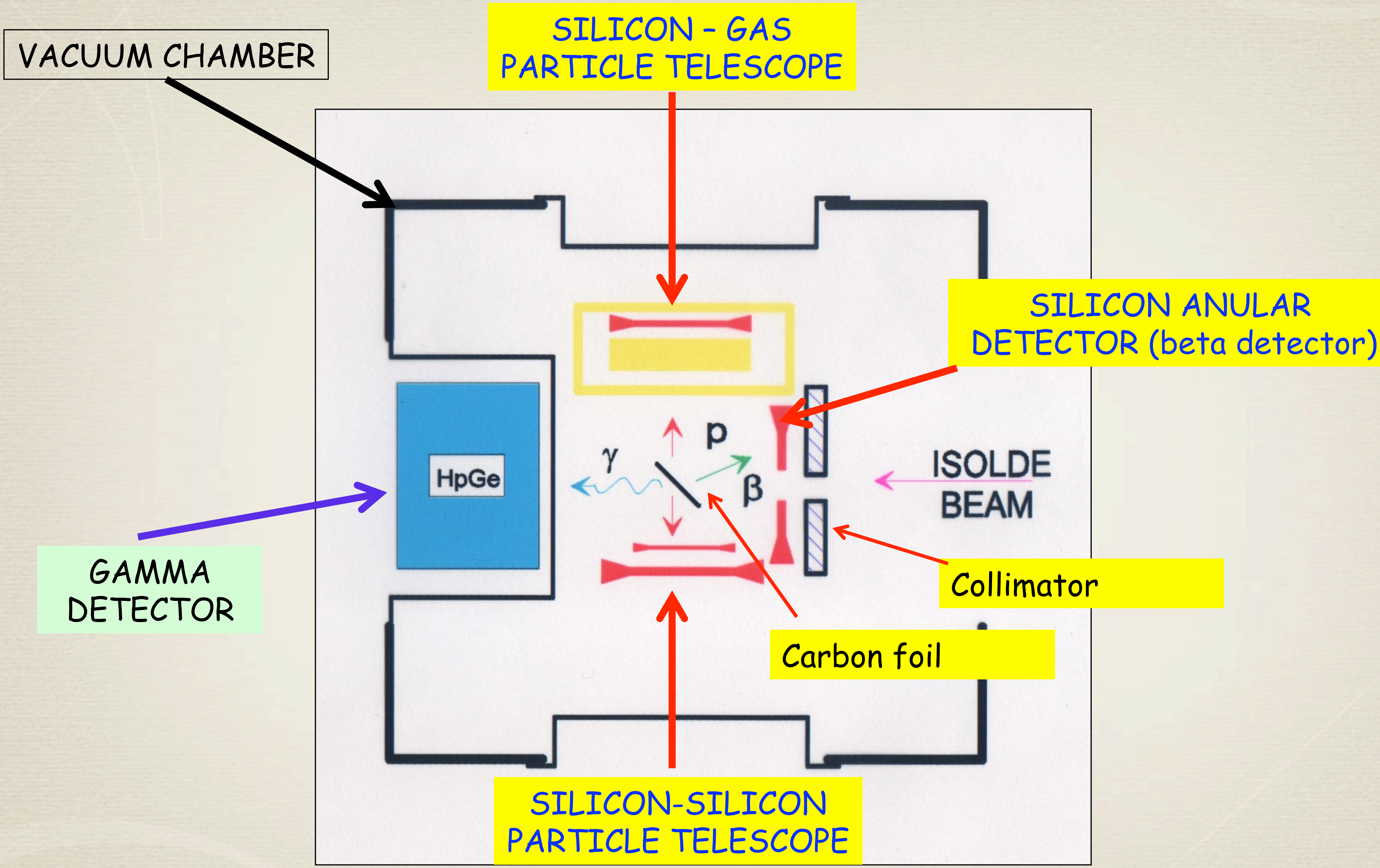




We need pure beams, free from isobaric contamination

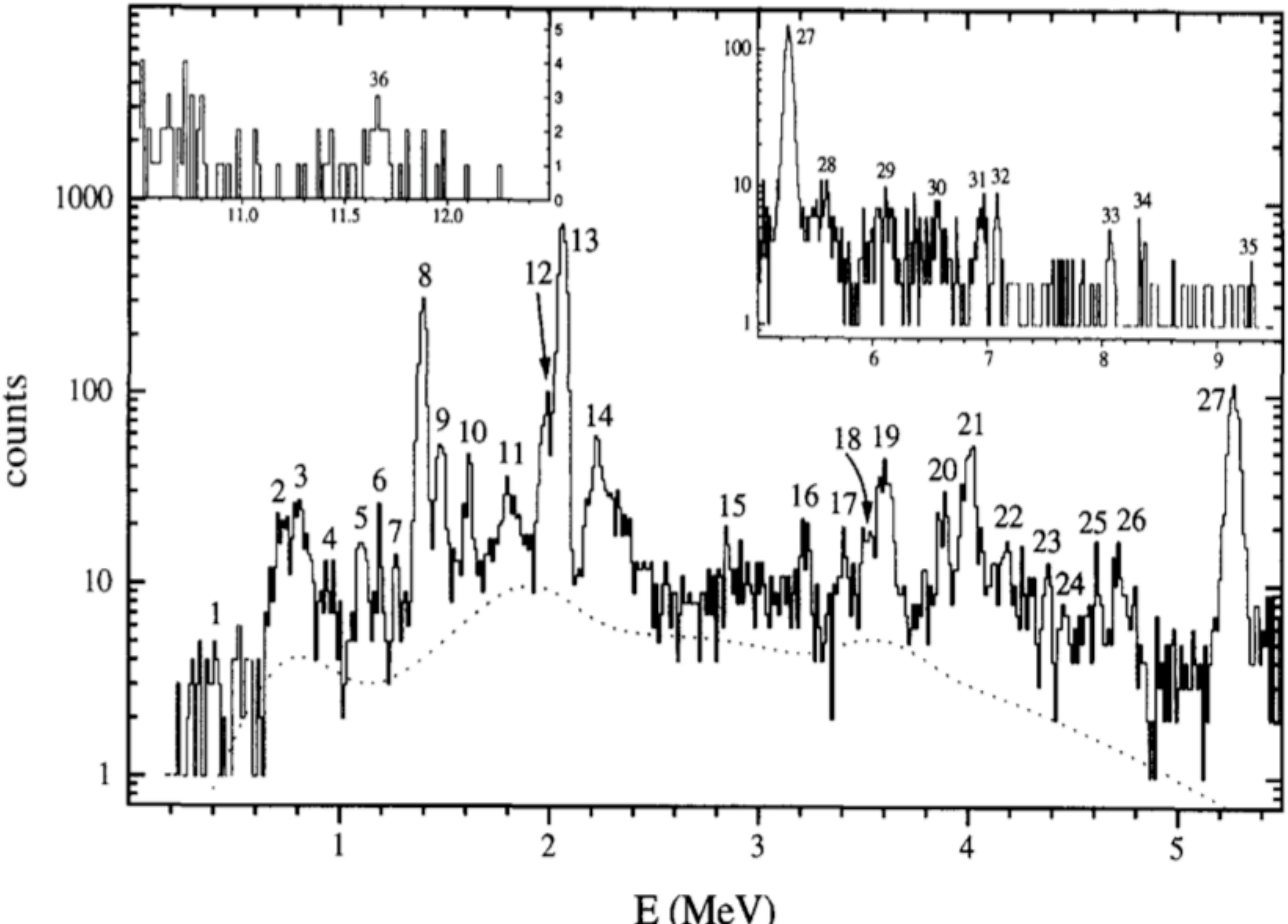
Example: ^{31}Ar produced at ISOLDE with a CaO-target and plasma ion-source (cooled transfer line) at a rate of 2 atom/s!!

TYPICAL EXPERIMENTAL SETUP

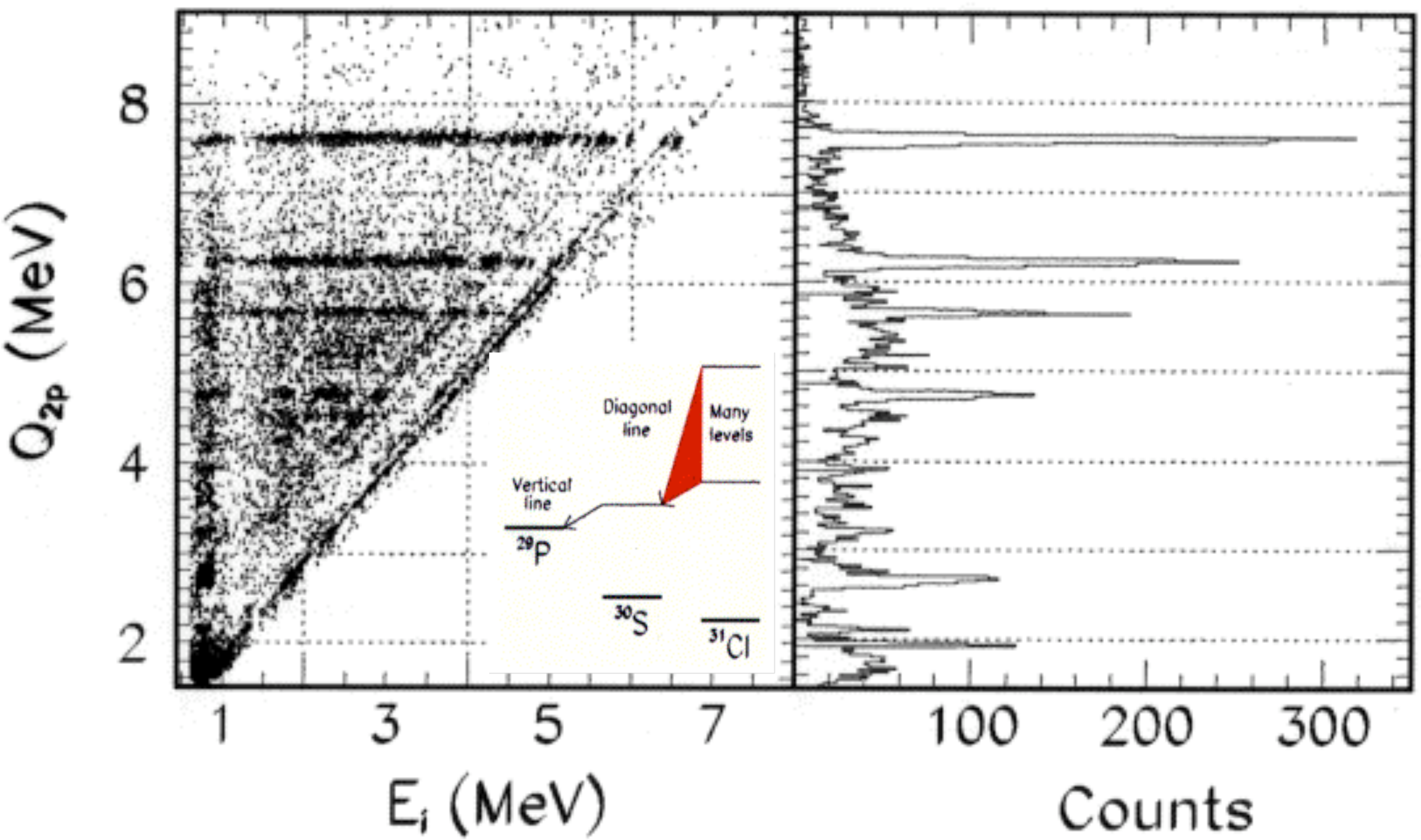


Low energy beam (~60 keV) → point like sources → good angular resolution → angular correlations

BETA DELAYED PROTON SPECTRUM



BETA DELAYED 2P EMISION FROM 31Ar
COINCIDENT P – P SPECTRUM



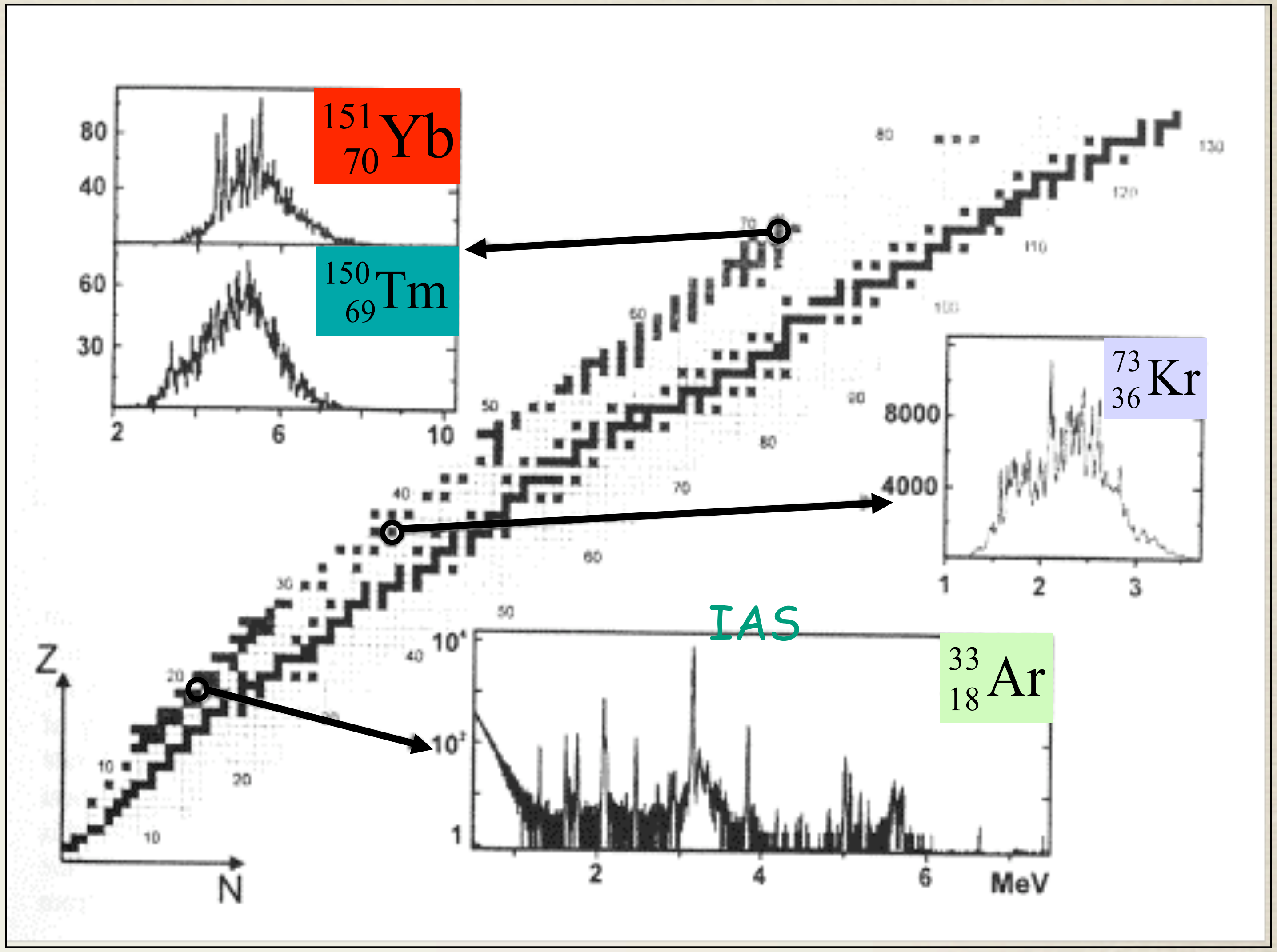
BETA DELAYED PROTON EMISSION (TODAY)

Today more than 134 precursor known

- Properties well understood
- This spectroscopic tool is often the only way to identify exotic nuclei
- Data provide large spectroscopic information:
  - Level density
  - Spin, isospin
  - Width & density
  - β-decay properties

- In ^{33}Ar ⇒ low level density, spectrum marked for proton peaks

- In the rest bellshape spectrum with superimpose peak structure ⇒ no individual transition rather cluster of them atributed to high density of states.



BETA DELAYED TWO PROTON EMISSION

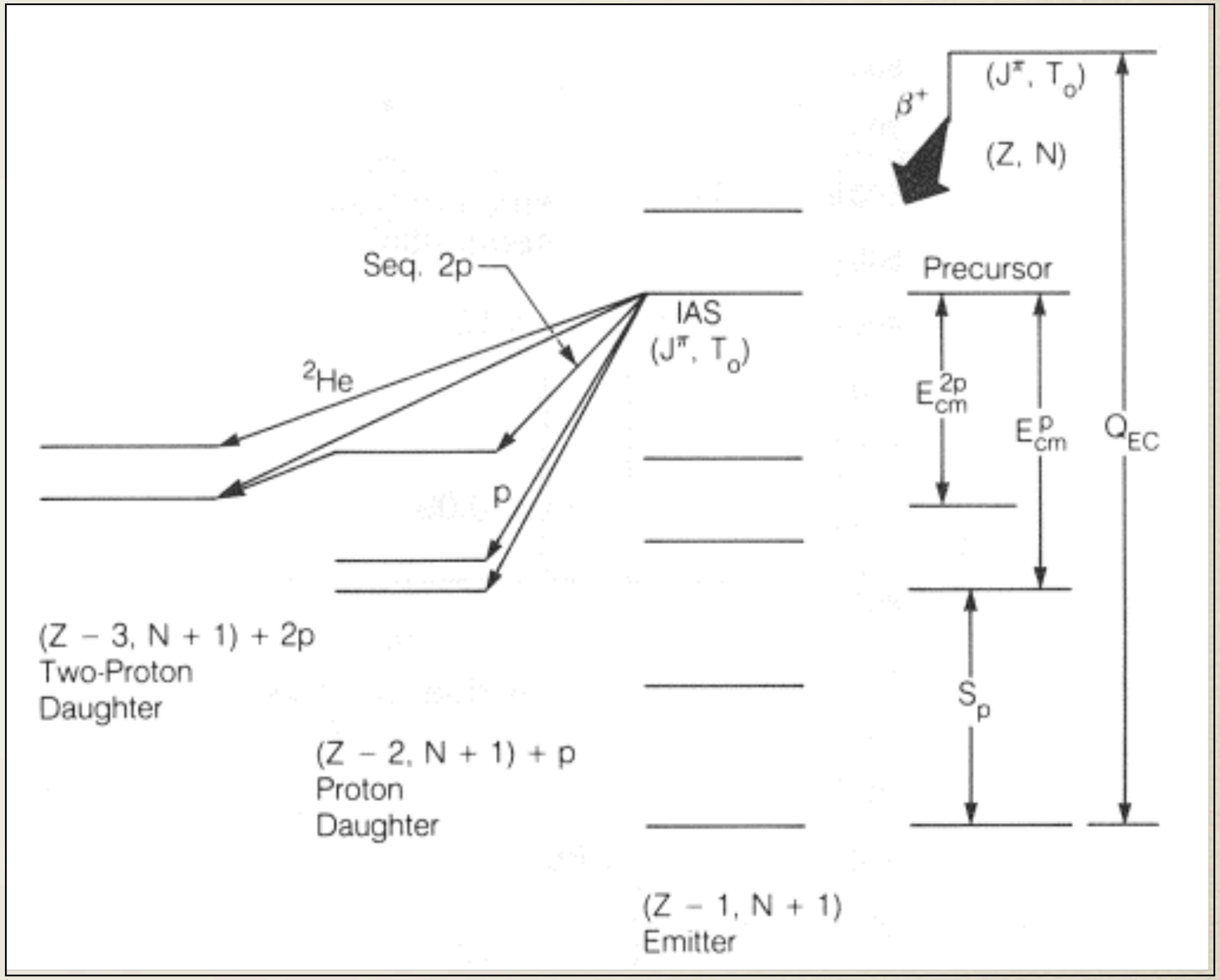
- Predicted in 1980 [*Goldanskii, JETP Lett. **32** (1980) 554*] as a mirror process of the β-2n branch detected in ^{11}Li

- This decay mode can proceed via three main mechanisms:

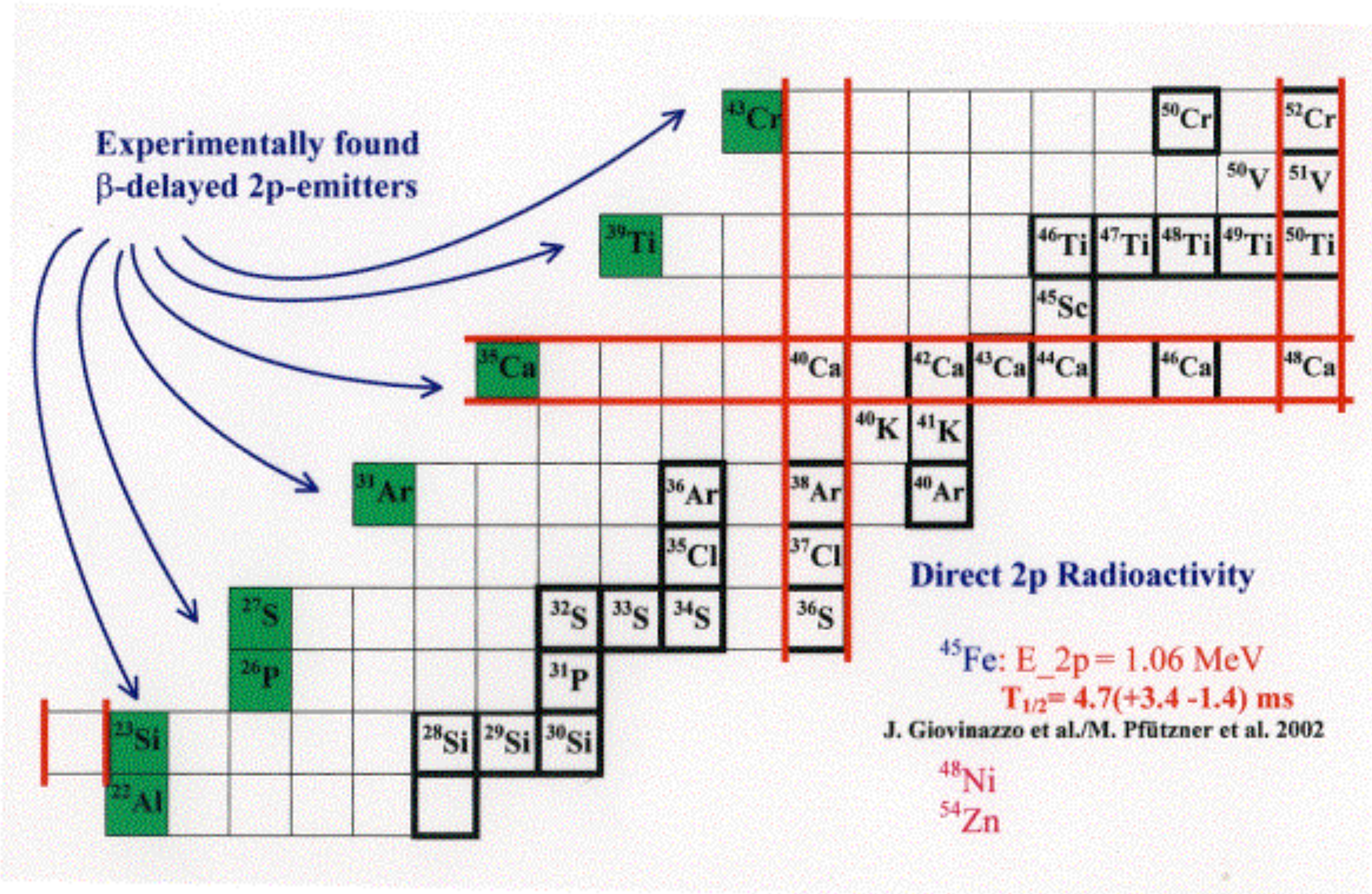
Sequential emission ⟺ β-p feeding to one or a few unbound states in the proton daughter nucleus. Two individual proton peaks, the second one broaden by the recoil of the proton daughter.
Low angular dependence.

"Di-proton" emission ⟺ simultaneous correlated emission. Broad individual peaks center at $E_{p1}=E_{p2}$
Narrow angular distribution

Democratic emission ⟺ where two body resonances do not play a significant role
Broad individual proton peaks.
Angular correlation dependence



Both the first and the last mechanism have been identified experimentally



Experimentally found
β-delayed 2p-emitters
43Cr
50Cr
52Cr
50V
51V
39Ti
46Ti
47Ti
48Ti
49Ti
50Ti
45Sc
35Ca
40Ca
42Ca
43Ca
44Ca
46Ca
48Ca
40K
41K
31Ar
36Ar
38Ar
40Ar
35Cl
37Cl
27S
32S
33S
34S
36S
26P
31P
23Si
28Si
29Si
30Si
22Al
Direct 2p Radioactivity
45Fe: E_2p = 1.06 MeV
T1/2= 4.7(+3.4 -1.4) ms
J. Giovinazzo et al./M. Pfützner et al. 2002
48Ni
54Zn

BETA DELAYED NEUTRON EMISSION

Nuclide	$T_{1/2}$ (ms)	P_{0n} (%)	P_{1n} (%)	P_{2n} (%)	P_{3n} (%)	P_{4n} (%)	References
^{11}Li	8.5(2)	6.3(6)	87.6(8)	4.2(4)	1.9(2)		Borge et al. PR C55 (1997) R8
^{14}Be	4.35(17)	14(3)	81(4)	5(2)			Dufour et al., PL B133 (1988) 146
			~100	$P_{2n}+3P_{3n}<2.4$			U.C. Bergmann et al., NP in press
^{17}B	5.08(5)	21(2)	63(1)	11(7)	3.5(7)	0.4(3)	Dufour et al., PL B133 (1988) 146
^{19}C	49(4)	46(3)	47(3)	7(3)			Dufour et al., PL B133 (1988) 146
^{30}Na	50(3)	69(4)	30(4)	1.17(16)			M. Langevin et al., NP A414 (1984) 151
^{31}Na	17.0(4)	62(5)	37(5)	0.9(2)			M. Langevin et al., NP A414 (1984) 151
^{32}Na	13.2(4)	68(7)	24(7)	8(2)			M. Langevin et al., NP A414 (1984) 151
^{33}Na	8.2(4)	36(21)	52(20)	12(5)			M. Langevin et al., NP A414 (1984) 151
^{34}Na	5.5(10)		$\beta(n+2n) = 115(20)$				M. Langevin et al., NP A414 (1984) 151

BASIC THEORY

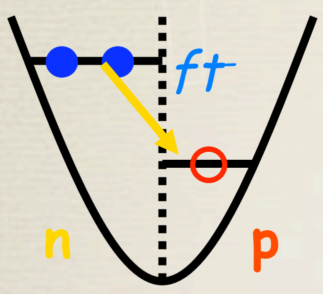
Beta decay

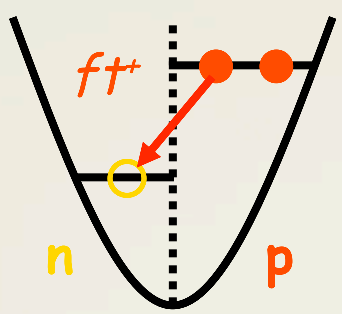
As it was previously discussed, weak interaction is one of the vehicles used for nuclear systems to release the excess of energy and travel from drip-lines to the Valley of Stability.

b^- : $n \rightarrow p + e^- + \nu$ b^+ : $p \rightarrow n + e^+ + \nu$

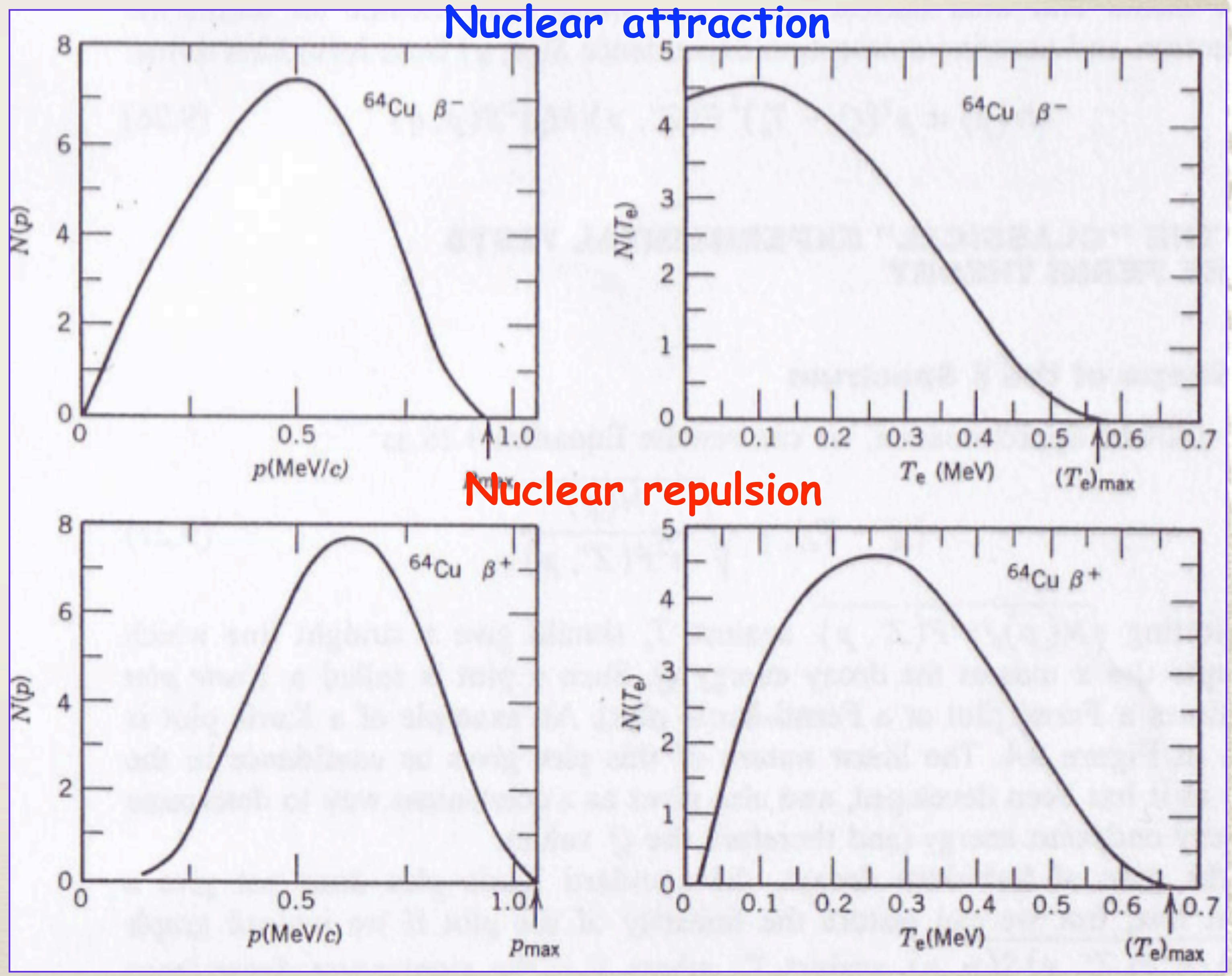
E.C. : $p + e^- \rightarrow n + \nu$

Competing process: **E.C.** Electron Capture, (Alvarez 1938)





Momentum & energy distributions



The energy spectrum of beta particles is continuous: **three body process**

Pauli 1931

- Neutrino
- Beta
- Residual nucleus

Neutrino → Reines & Cowan 1950

K.S. Krane, Intr. Nucl. Phys., John wiley & Sons, 1988

Neutron decay b^- : $n \rightarrow p + e^- + \nu$ About 10 minutes!!

$Qb = (mn - mp - me^- - m\nu)c^2 = Tp + Te + T\nu$ == measured to be 0.782 +/- 0.013 Kev

mn = 939.573 MeV
mp = 938.280 MeV
me^- = 0.511 MeV

$m\nu$ ~13 eV???? or Zero???
→ can assume $m\nu = 0$

Beta- decay

$X(A,Z,N) \rightarrow X(A, Z+1,N-1) + e^- + \nu$

$Qb^- = (Mn(A,Z,N) - Mn(A, Z+1,N-1) - me^-)c^2$ Nuclear masses

$Ma(A,Z,N) = Mn(A,Z,N) + Z\, me^-)\, c^2 - \Sigma B(i)$ Atomic masses

$Qb^- =\sim (Ma(A,Z,N) - Ma(A, Z+1,N-1))c^2$ Impact of atomic mass measurements on Qvalue determination (or inverse!)

Beta+ decay

$X(A,Z,N) \rightarrow X(A, Z-1,N+1) + e^+ + \nu$

$Qb^- = (Ma(A,Z,N) - Ma(A, Z-1,N+1) - 2\, me^-)c^2$ Electron mass do not cancel!!

EC decay

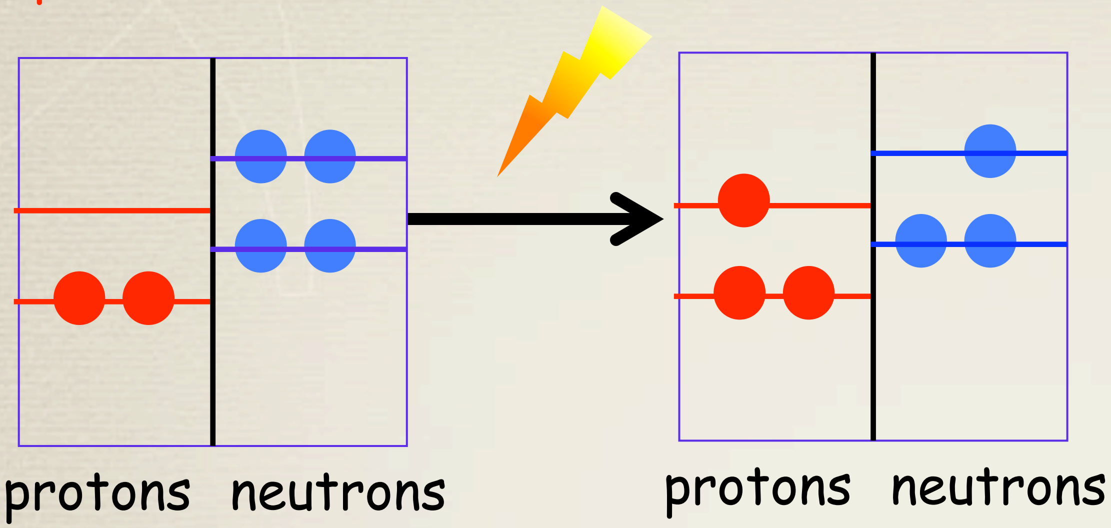
$X(A,Z,N) + e^- \rightarrow X^*(A, Z-1,N+1) + \nu$ Atomic excited state ~Bn

$Qb^- = (Ma(A,Z,N) - Ma(A, Z-1,N-1) - 2\, me^-)c^2 + Bn$

EC is always accompanied by b+, but not the opposite, in general.

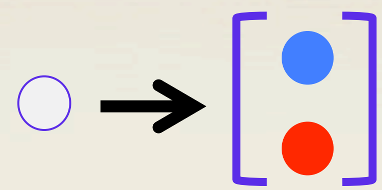
Beta decay and isospin

Beta decay: transformation of proton ←→ neutron



Concept of **nucleon**: particle that can be proton/ neutron:

→ new quantum number ISOSPIN (T) describes "the charge state of the nucleon"

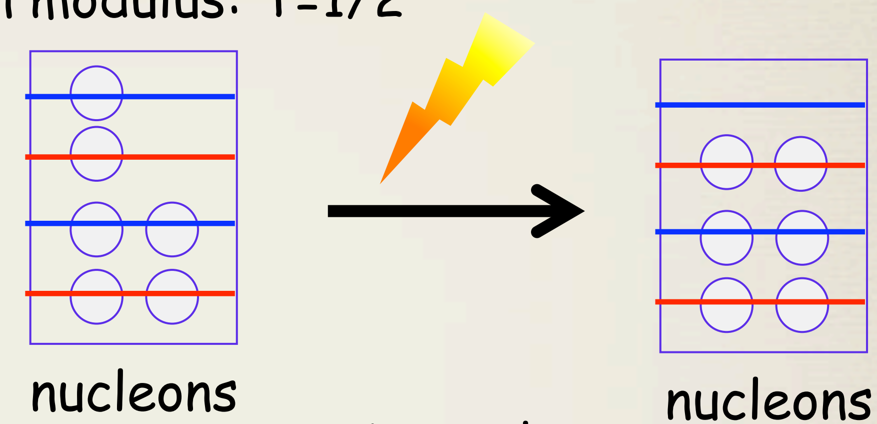


Dirac ket

$T_z=-1/2$ (neutron) $|T=-1/2\rangle$

$T_z=+1/2$ (proton) $|T=+1/2\rangle$

Isospin modulus: T=1/2



eigenvalues

$T_z\,|T=+1/2\rangle = +1/2\,|T=+1/2\rangle$ C

$T_z\,|T=-1/2\rangle = -1/2\,|T=+1/2\rangle$

OPERATIONS & OPERATORS

$|T=-1/2\rangle$ neutron $|T=+1/2\rangle$ proton

$I = |T=-1/2\rangle\langle T=-1/2| + |T=+1/2\rangle\langle T=+1/2|$, identity

$\langle T=-1/2|T=+1/2\rangle = \langle T=-1/2\,|T=+1/2\rangle = 0$, orthogonal

$\langle T=-1/2|T=-1/2\rangle = \langle T=+1/2\,|T=+1/2\rangle = 1$, normalization

$T_z = (-1/2)\,|T=-1/2\rangle\langle T=-1/2| + (+1/2)\,|T=+1/2\rangle\langle T=+1/2|$ The isooospin operator

$T_+ = |T=+1/2\rangle\langle T=-1/2|$

$T_- = |T=-1/2\rangle\langle T=+1/2|$

Isospin flip operators

$T_+\,|T=-1/2\rangle$ = neutron → proton = $|T=+1/2\rangle$

$T_-\,|T=+1/2\rangle$ = proton→ neutron = $|T=-1/2\rangle$

$T^{+/-}$ → beta decay operators!

$Q = (2T_z + 1)/2$ = charge operator

$$M(A,T,T_z) = a(A,T) + b(A,T)\,T_z + c(A,T)\,T_z^2$$

The isobaric multiplet mass equation (IMME), Wigner 1957

→ drip lines, exotic radioactivity, etc

For a system nucleons $T = \Sigma\, T(i)$ $T_z = \Sigma\, T_z(i)$ $T^{+/-} = \Sigma\, T^{+/-}(i)$ i=1…A nucleons

Beta interaction $Vb^{(+/-)} = g_V\, T^{(+/-)} + g_A\, ST^{(+/-)}$

g_V = Fermi constant (vector constant)
g_A= Gamow-Teller constant (axial-vector constant)

Experimentally, beta decay can change spin of final nuclei (S operator)

Beta transition prob. → Fermi Golden Rule

$$\lambda\,(i, f; En, Eb) = 2\pi/\hbar\, |\langle\, i\, |Vb|\, f\, \rangle|^2\, \rho\,(Q-En, Eb)$$

Transition probability

matrix element

Density of final states β , ν recoil excit: En

$$|M(F)|^2 = |\langle\, i\,|T^{(+/-)}\,|f\,\rangle|^2$$
$$|M(GT)|^2 = |\langle\, i\,|ST^{(+/-)}\,|f\,\rangle|^2$$

$$\rho\,(Q-En, Eb) \sim p_b (Q-Eb)^2\, F(Zf, Eb)$$

Phase space

nucl rep./atrac. (Fermi function)

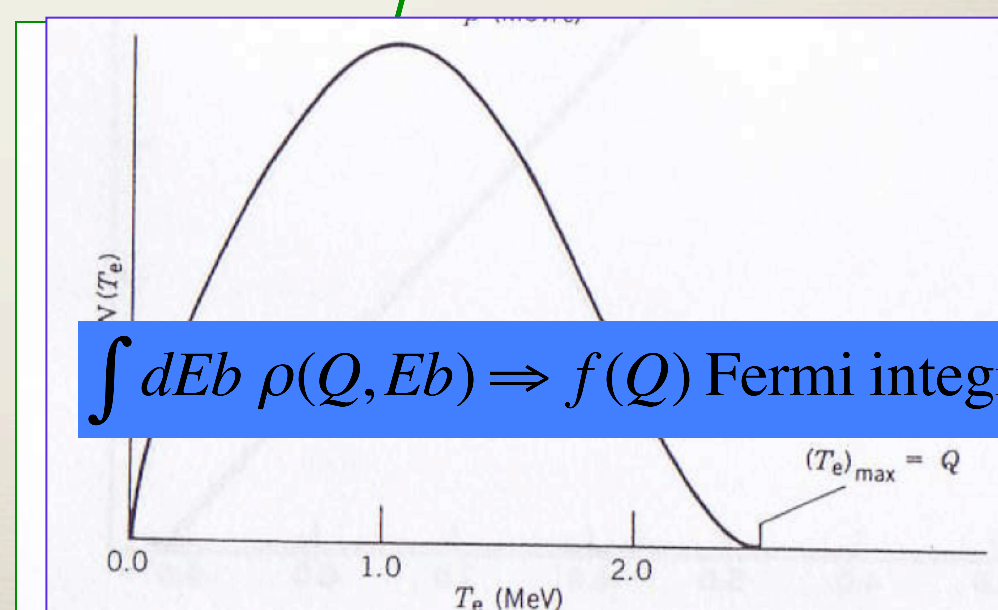
$| i \rangle = |A; i \rangle$ = initial nuclear state

$|A; f \rangle = |A; f \rangle \times |\beta \rangle \times |\nu \rangle$ final state

1st Forbidden decays, etc

$$|\beta\rangle x |\nu\rangle \sim e^{ik_\nu r} e^{ik_\beta r} \sim 1-(k_\beta + k_\nu)r + \ldots \sim 1$$

Allowed aproximation
Allowed decays
L=0 (r=0)



$$\int dEb\, \rho(Q, Eb) \Rightarrow f(Q) \text{ Fermi integral}$$

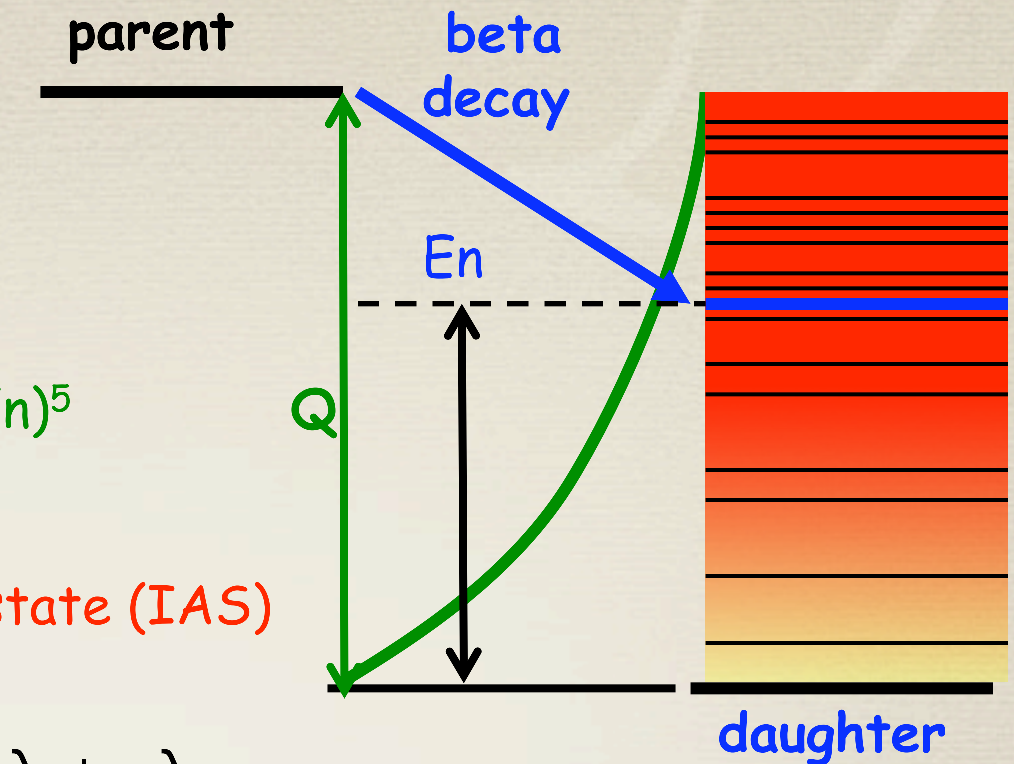
$$(Eb+m_e c^2)\,(Eb+2Eb m_e c^2)^{1/2}\,(Q-Eb)^2$$

Finally

$$\lambda(i,f;En) = \frac{m_e^5 c^4}{2\pi\hbar^7}\left[g_V^2 |M(F)|^2 + g_A^2 |M(GT)|^2\right] f(Z_f, Q-En)$$

$$f(Z,E) = \frac{1}{m_e^5 c^7} \int_0^{p\max} F(Zf,p)\, p^2 (E-Eb)\, dp \quad \text{Fermi integral}$$

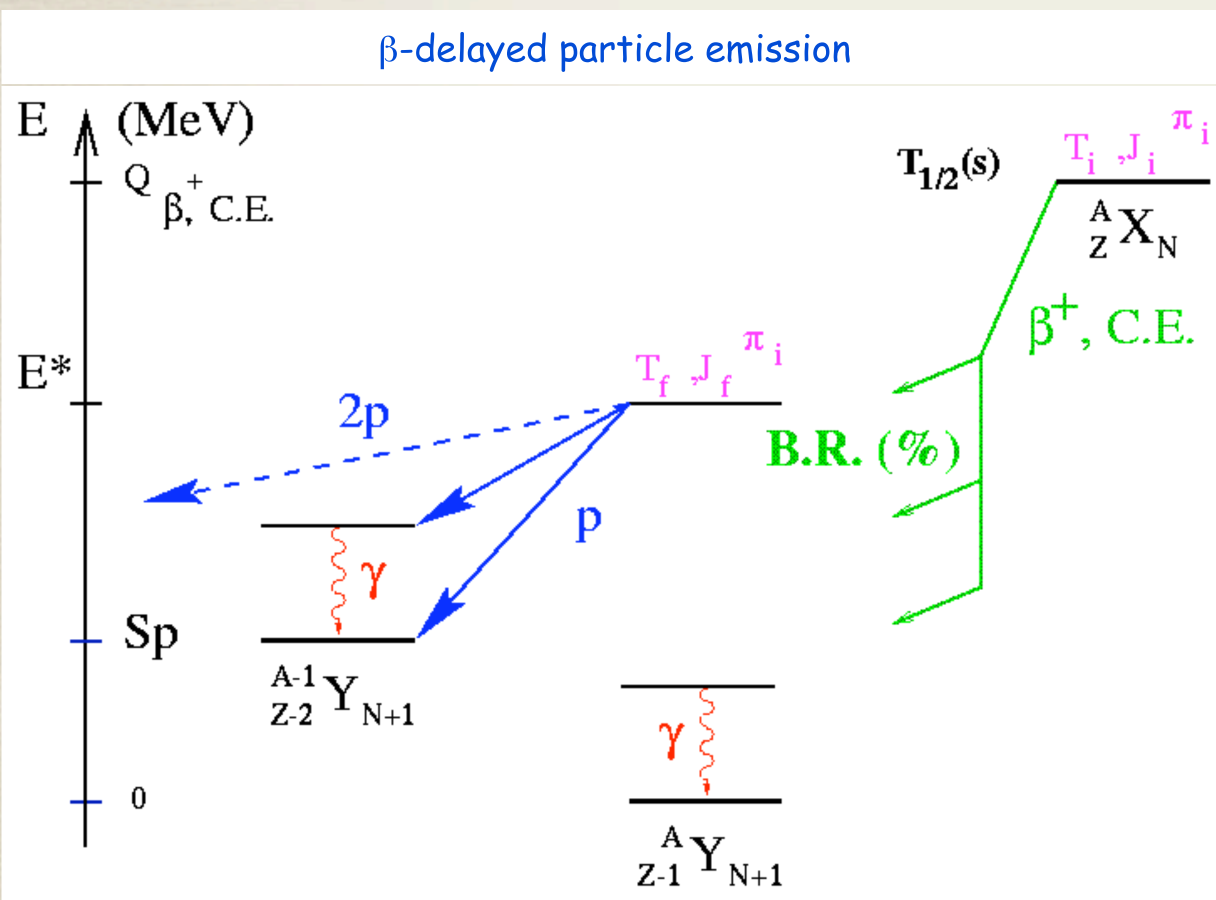
$F(Z_f,Q\text{-}En) \sim (Q\text{-}En)^5$



- Selection rules for allowed approx:
  - Fermi: $\Delta T=0$; $\Delta J=0$; $\pi_f = \pi_i$ → Isobaric analog state (IAS)
  - Gamow-Teller: $\Delta T=0\pm1$; $\Delta J=0\pm1$; $\pi_f = \pi_i$

Branching ratios and partial half-life

- $\lambda_T=\lambda_1+\lambda_2+\ldots\lambda_N$
- $T_T= Ln(2)/\lambda \rightarrow 1/T=1/T_1+1/T_2\ldots1/T_N$
- $Br(i)= \lambda_i/\lambda_T=T_T/T_i$



ft-value (comparative half-life)

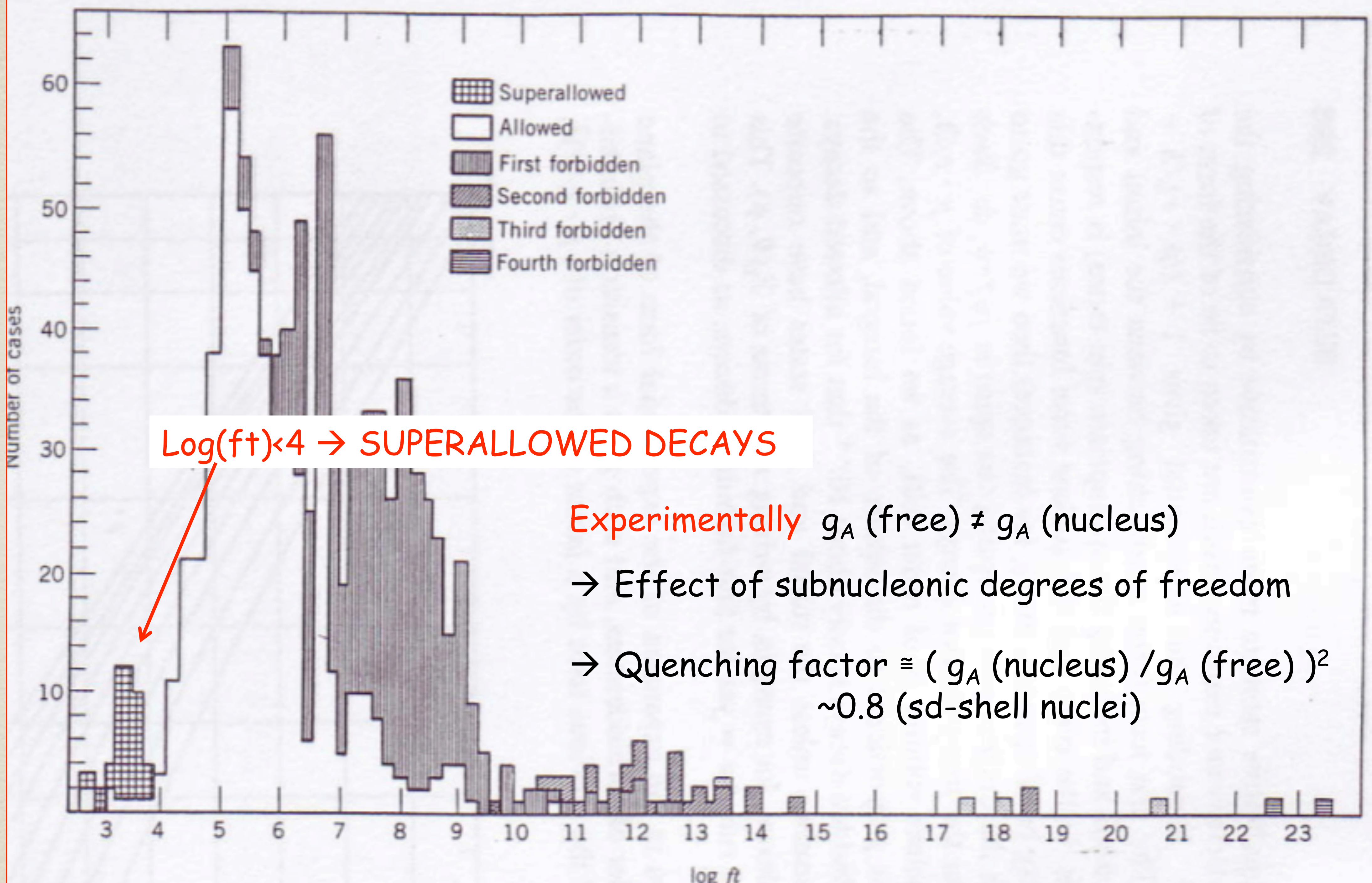
$$\lambda = Ln(2)/T_{1/2}$$

$$ft = f * \frac{T_{1/2}}{Br} = \frac{K}{g_V^2 |M(F)|^2 + g_A^2 |M(GT)|^2}$$

$$ft = \frac{C}{B(F)+B(GT)}$$

B(F), B(GT): reduced transition probability

Large range ft~ 10^3 → 10^{20} →Tabulate Log(ft)



Log(ft)<4 → SUPERALLOWED DECAYS

Experimentally g_A (free) ≠ g_A (nucleus)

→ Effect of subnucleonic degrees of freedom

→ Quenching factor ≅ (g_A (nucleus) / g_A (free) $)^2$ ~0.8 (sd-shell nuclei)

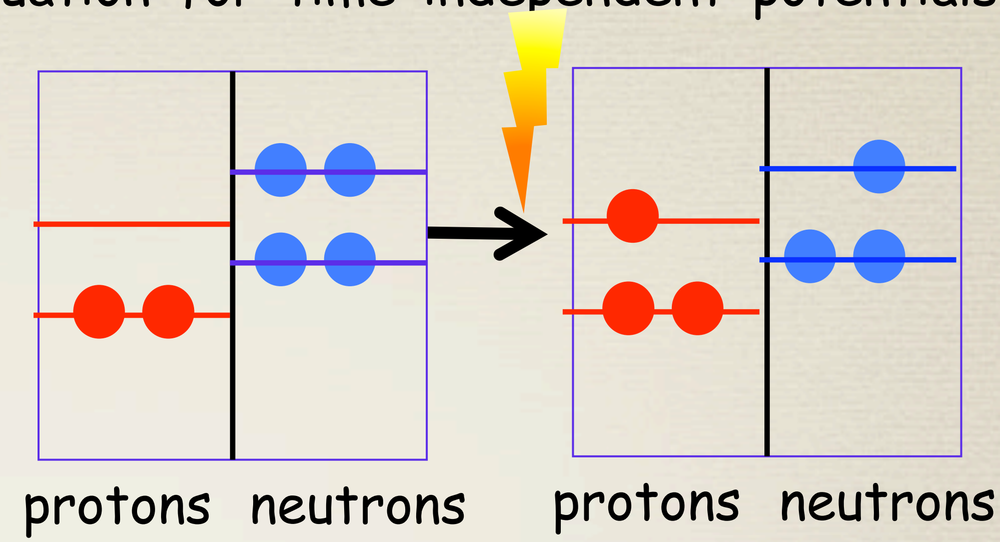
Figure 9.9 Systematics of experimental log *ft* values. From W. Meyerhof, *Elements of Nuclear Physics* (New York: McGraw-Hill, 1967).

K.S. krane, “Introductory Nuclear Physics”; John Wiley & Sons, NY, 1988

Particle emission: transitions and decaying states

The wave functions obtained by solving the Schrödinger equation for time independent potentials have the property of being stationary states

$$\hat{H}\Psi_o(r,t) = E(0)\Psi_o(r,t) \qquad \Psi_o(r,t) = \Psi_o(r)e^{-i\frac{E(0)}{\hbar}t}$$



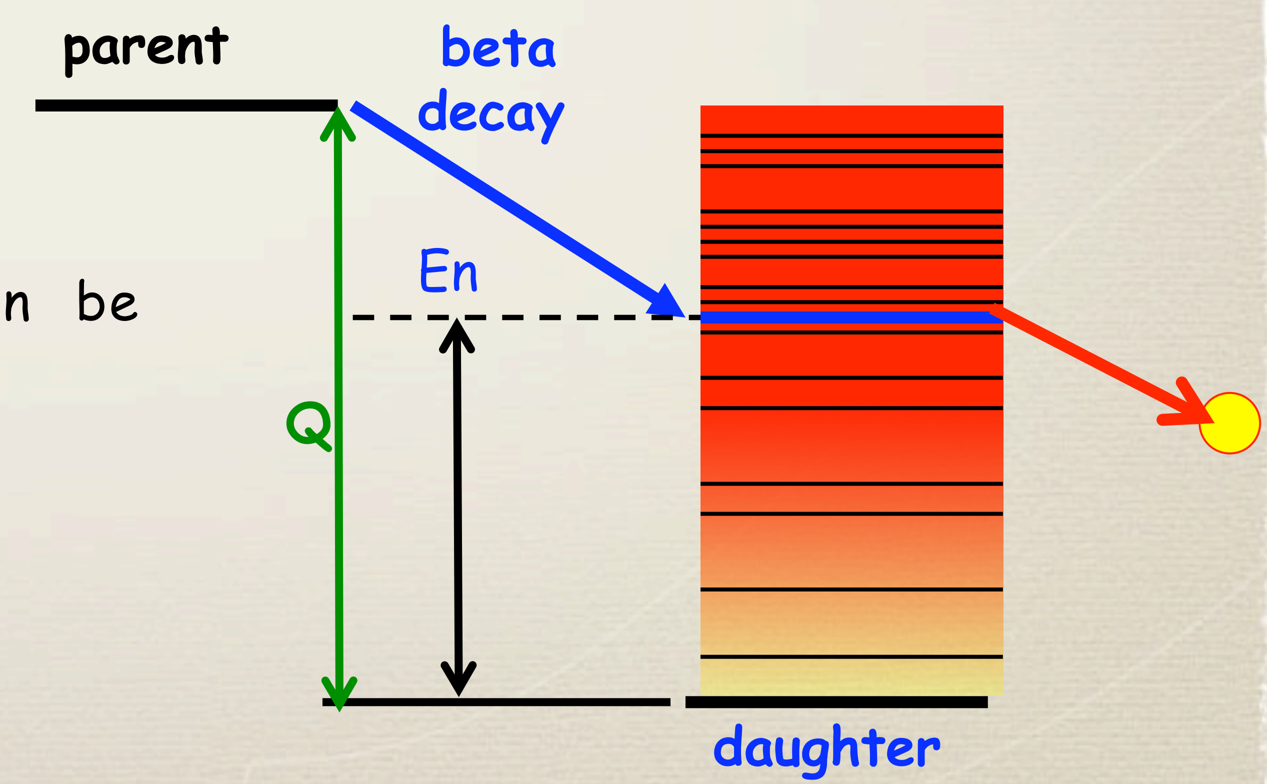
States will remain in that energy eigenstate forever!

Under a sudden change of the potential (like beta decay p←→n), we get a new hamiltonian H_{new} and the "old" wavefunctions are no more eigenvalues → start evolution with time:

$$\hat{U}(t)\Psi_o(r,t) = e^{-i\frac{\hat{H}_{new}}{\hbar}t}\Psi_o(r,t) = \sum c_i(t)\phi_i(r)e^{-i\frac{E(i)_{new}}{\hbar}t}$$

$$\hat{H}_{new}\phi_i(r,t) = E(i)_{new}\phi_i(r,t)$$

Some of these new states are continuum states → particle emission



The transition process (particle emission) can be described by the Fermi Golden Rule

$$\lambda = \frac{2\pi}{\hbar}\left|V_{if}\right|^2 \rho(E_f)$$

$$V_{if} = \int \Psi_f^*(H_{new} - H_{old})\Psi_i \qquad \rho(E_f) = \frac{dn_f}{dE_f}$$

If Ef > Sp → tunnel through Coulomb barrier
→ P(r,t) decreases with time.

→ use of a complex energy eigenvalue in the final system:

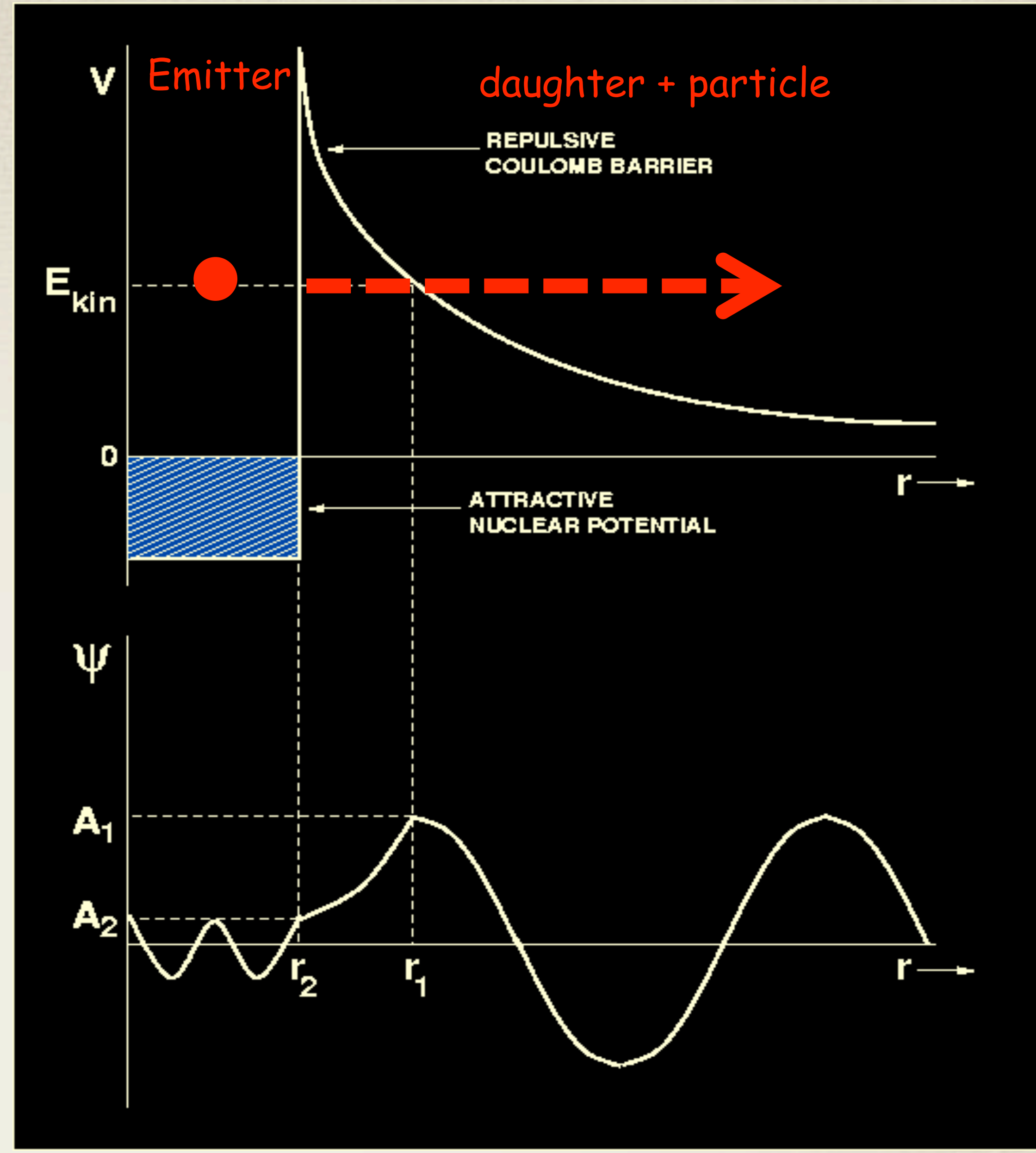
$$E_d + i\,\Gamma_d/2$$

$$\phi_d(r,t) = N\phi_d(r)e^{-\frac{i}{\hbar}(E_d+i\Gamma_d/2)t} = N\phi_d(r)\,e^{-\frac{i}{\hbar}E_d t}\,e^{\frac{1}{\hbar}\frac{\Gamma_d}{2}t}$$

$$P(r,t) = N^2\left|\phi_d(r,t)\right|^2 = N^2\left|\phi_d(r)\right|^2 e^{-\frac{1}{\hbar}\Gamma_d t}$$

$$P(r,t) = N^2\left|\phi_d(r)\right|^2 e^{-\lambda_d t}$$

$$\lambda = \frac{1}{\tau} = \frac{\Gamma}{\hbar}$$

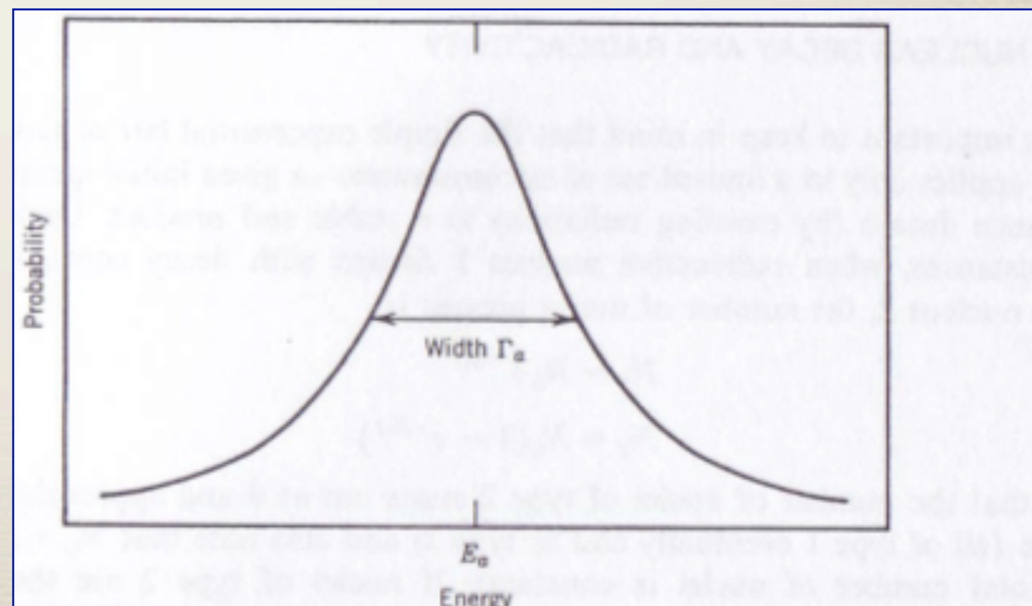


For the energy distribution (energy representation)
→ Fourier transform

$$\phi_d(E) \approx \int e^{-\frac{i}{\hbar}Et}\phi_d(t)\,dt \approx \int e^{-\frac{i}{\hbar}Et}\,e^{-\frac{i}{\hbar}E_d t}\,e^{-\frac{1}{\hbar}\frac{\Gamma_d}{2}t}\,dt$$

$$\phi_d(E) \approx \frac{1}{(E-E_d)+i\frac{\Gamma}{2}}$$

$$P(E) \approx \frac{1}{(E-E_{dec})^2 + \left(\frac{\Gamma}{2}\right)^2}$$



3 Probability to observe the energy of an unstable state of width Γ_a.

Why complex eigenvalues? → $E_d + i\,\Gamma_d/2$ → naturally arise from solving Shrödinger ecuation at E>0!

Georg Gamow : simple model of alpha decay, G.A. Gamow, Zs f. Phys. 51 (1928) 204; 52 (1928) 510

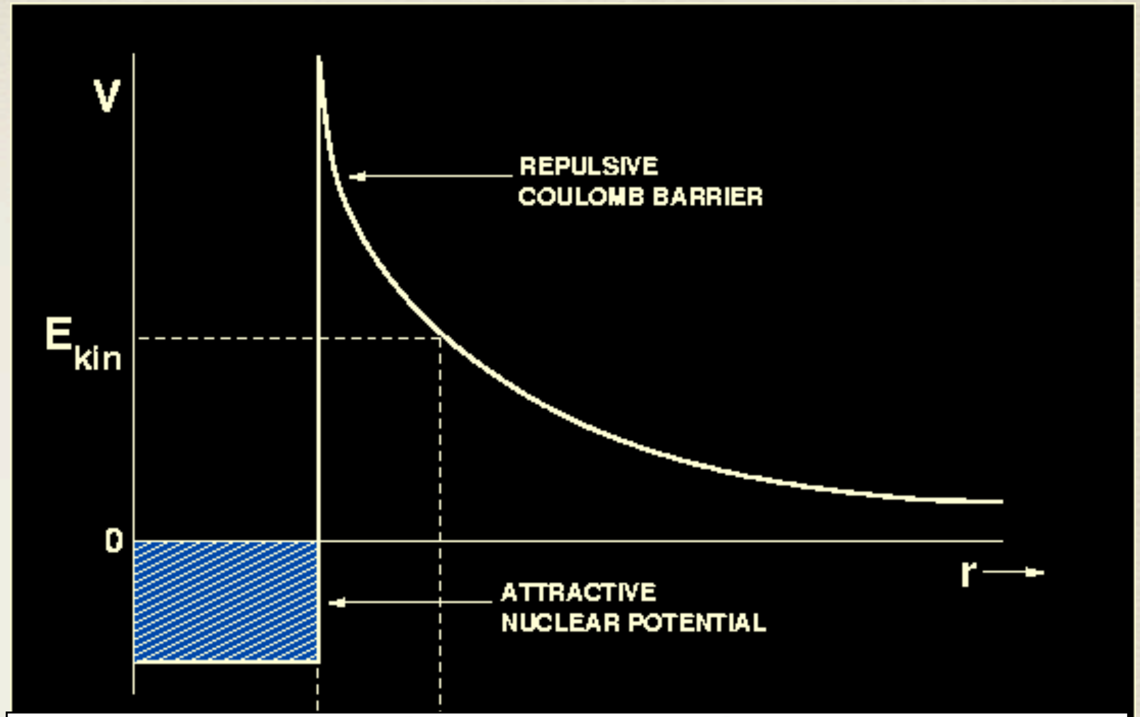
→ Quantum tuneling through barrier

$$u''(r) = \left[\frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}V(r) - k^2\right] u(r)$$

$$u(r) \sim C_0\, r^{l+1} \;,\; r \to 0$$

$$u(r) \sim C_+\, H^+_{l,\eta}(kr) \;,\; r \to +\infty \text{ (bound,resonant)}$$

$$u(r) \sim C_+\, H^+_{l,\eta}(kr) + C_-\, H^-_{l,\eta}(kr) \;,\; r \to +\infty \text{ (scattering)}$$



If keep same boundary condition → $H^+(kr)$, r→ ∞
Bound and resonant states → poles of the Scattering matrix S(k) (matching with outgoing WF)

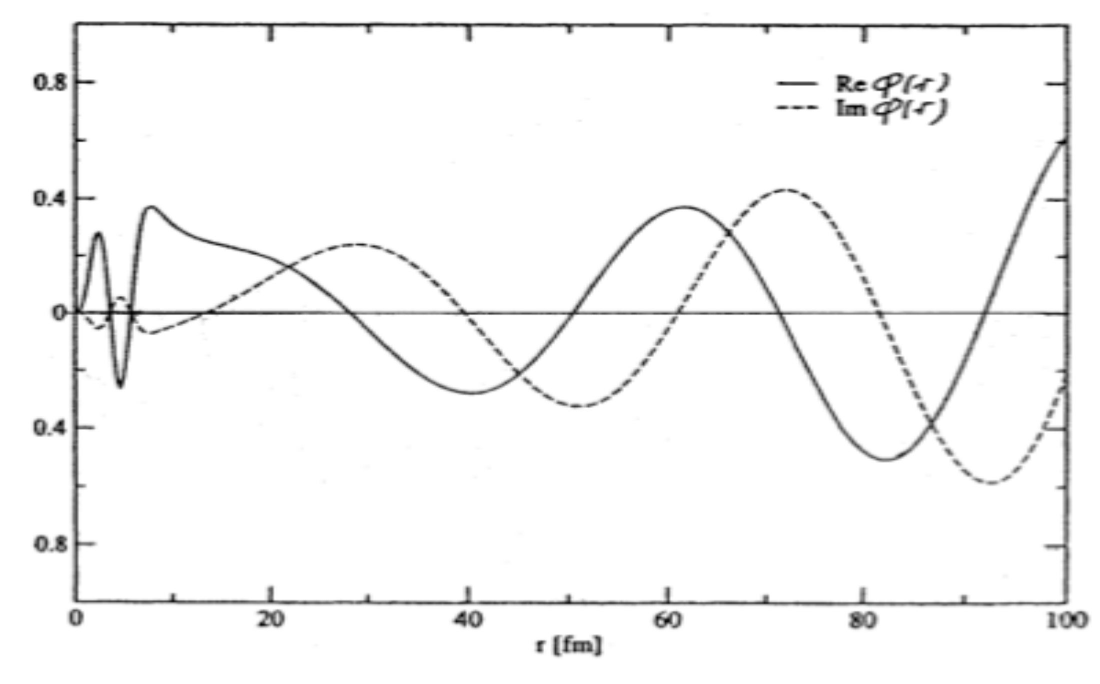


Figure 1: Gamow radial wave function $\varphi_{nl}(r)$

Bound states:

→ pure imaginary K values: ~ - i Ki, Er < 0

Resonant states:

→ complex K values: Kr – i Ki, Er > 0, Γ> 0

→ GAMOW STATES

$$\hat{1} = \sum_{i=b} |u_i\rangle\langle\tilde{u}_i| + \sum_{j=r} |u_j\rangle\langle\tilde{u}_j| + \int_{L^+} |\varphi(k)\rangle dk \langle\tilde{\varphi}(k^*)|$$

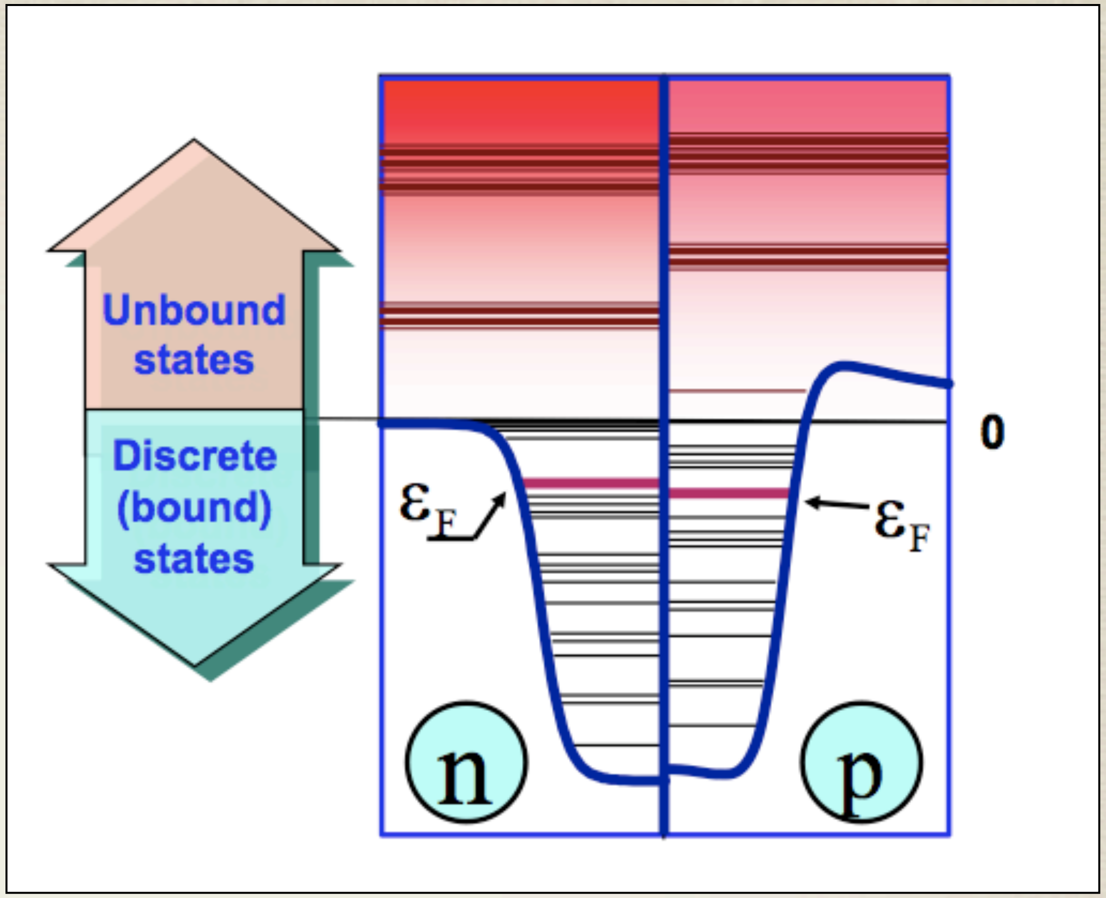
Consistent description of bound and scattering states: → a rigged Hilbert space (Gel'fand triple space): 1960s Gel'fand combined Hilbert space with the theory of distributions.

Spectacular applications: Shell model in the continuum// → Shell model in the complex energy plane; N. Michel, W. Nazarewicz, M. Oloszajzak and T. Vertse (J. Phys. G.: Nucl. Part. Phys. 36 (2009) 013101

Difficult to overestimate the importance of Gamow theory!!.

Some references: Humblet and Rosenfeld, Nucl. Phys. 26, 529 (1961); T. Berggren, Nucl. Phys. A 109 (1968) 265. R. de la Madrid, Nucl. Phys. A812, 13 (2008)

Gamow states of a finite potential



R-MATRIX DESCRIPTION

Tradicional method → based on R-matrix theory for unbound nuclei → scattering, reactions, particle decay. (F. C. Barker, Aust. J. Phys., 1988, 41, 743-63, E.K. Warburton, PRC 33 (1986)303-313)

$$P(E) \propto \left| \sum_i \frac{G(i)^{1/2}\Gamma(i)^{1/2}}{\left(E(i)+\Delta(i)-E-i\frac{\Gamma}{2}\right)} \right|^2$$

G(i) : feeding factor of the decaying state

Γ(i) : level width **Γ(i) : 2 P(E)*γ²**

Δ(i) : shift factor (WF matching at pot. radius)

E(i) : level energy/resonance

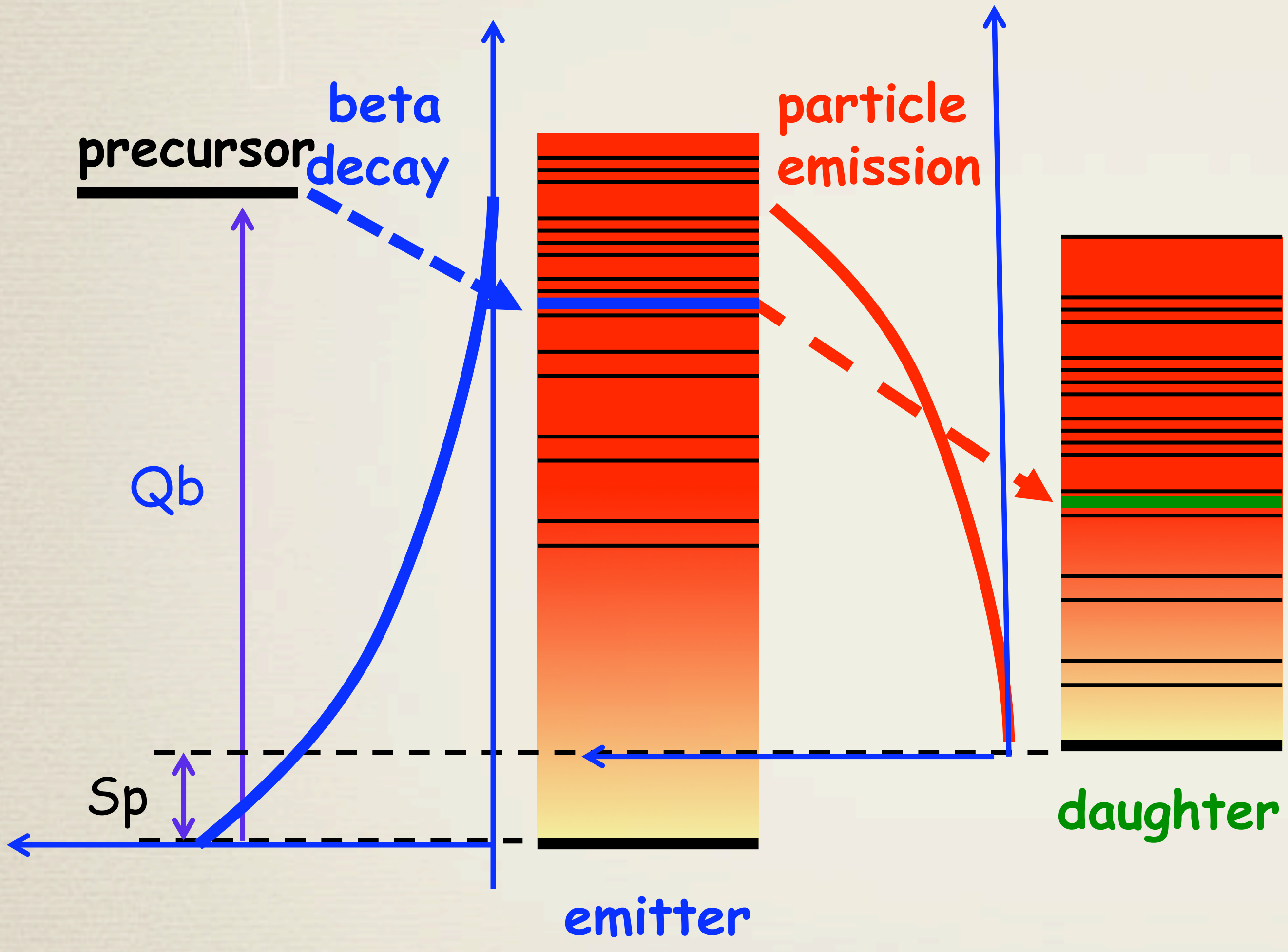
Reduced width (nuclear matrix element)

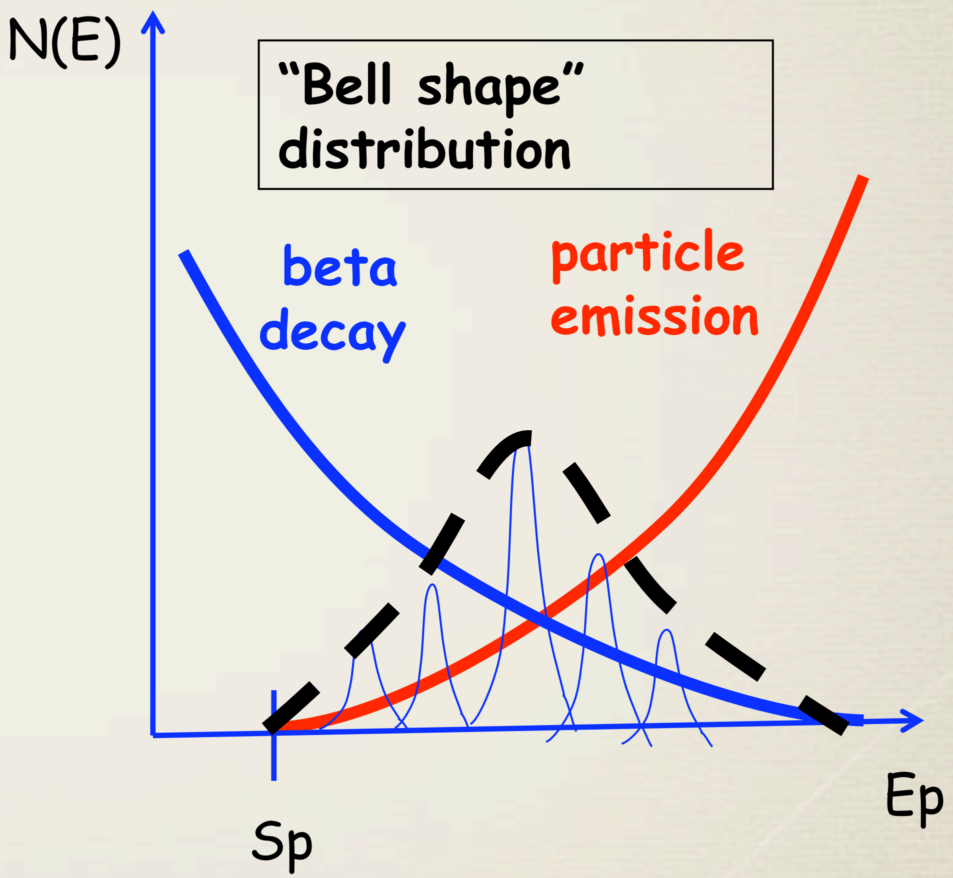
Penetration factor (barrier)

SUMMARY: what to expect for beta delayed particle emission

Two processes:

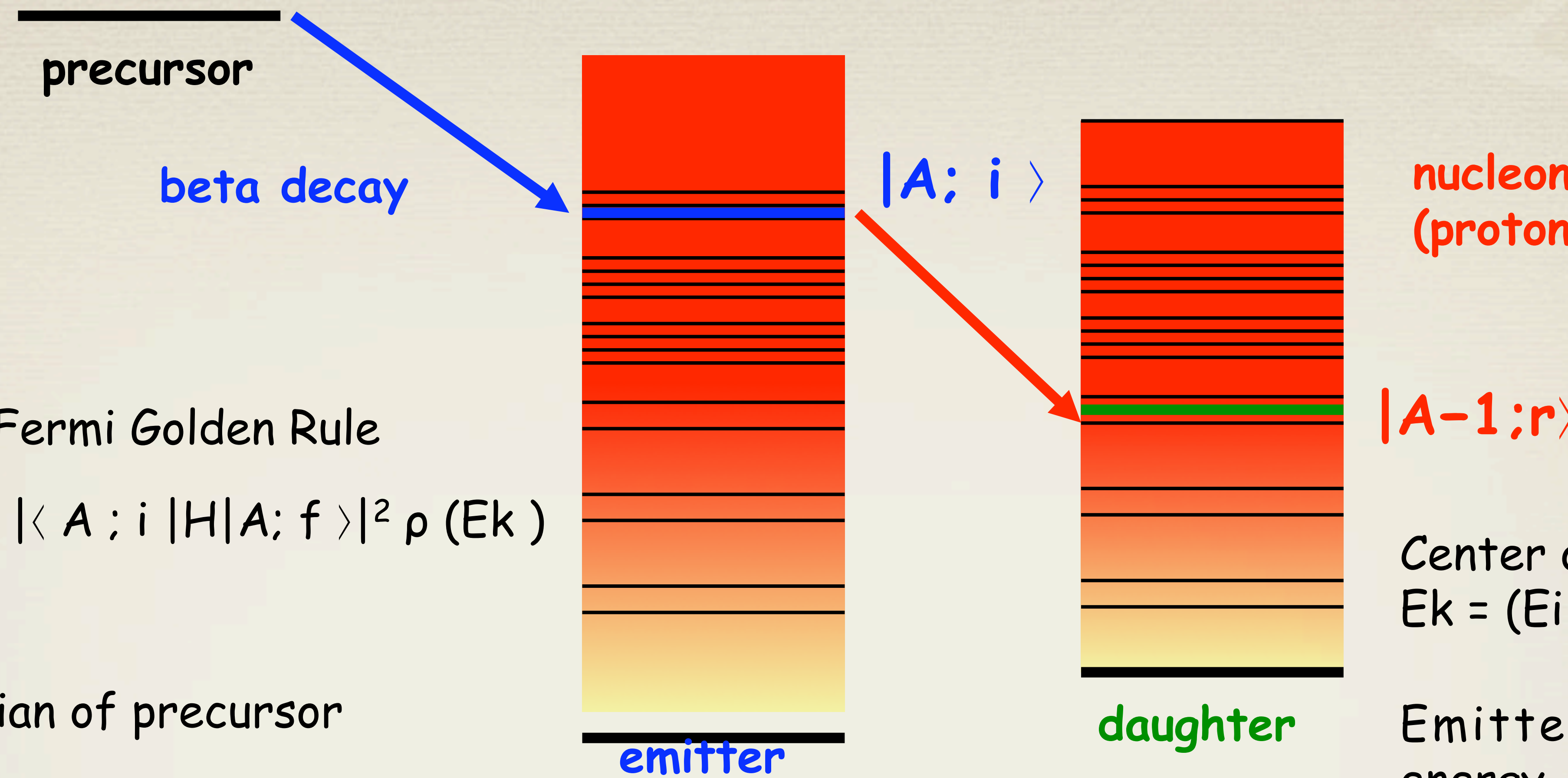
- Beta decay → FERMI INTEGRAL (Matrix elements) → $(Q\text{-}E_n)^5$
- Particle emission → BARRIER PENETRABILITY ~ P(Ek) ~ 1/(1+ exp((EB–Ek)/ωb) (parabolic)
- Breit – Wigner shapes on each level
- Density of states above Sp





A BASIC EXAMPLE

Simple model for beta delayed nucleon emission



Cooking recipe:

Decay width → Fermi Golden Rule

$\Gamma(i, f, Ek) = 2\pi\, |\langle A ; i |H|A; f \rangle|^2 \rho(Ek)$

Ingredients:

H: full Hamiltonian of precursor

Center of mass energy
$Ek = (Ei + Q_N - Er)$

Emitted particle energy
$Ep = (A - 1/A)\, Ek$

$|A; i\rangle$: Initial state; eigenstate of H in a shell-model space, constructed from single-particle wave functions.

$|A; f\rangle$: Final state, one of the nucleons in a single-particle state @ continuum.

$\langle A ; i |A; f \rangle = 0$ (orthogonal)

$|A, i \rangle = \Sigma_{rnlj}\, SPA(i ; r, nlj)\, |A - 1; r \rangle \times |N(B), nlj\rangle$

$|A, f \rangle = |A - 1; r \rangle \times |N(U), Eklj\rangle$

- $|N(B), nlj\rangle$: single-particle wave functions (shell-model)
- SPA(i; r, nlj): spectroscopic amplitudes
- $|N(U), Eklj\rangle$: single-particle positive-energy state lying in the continuum

I. Martel et al., NPA(2001)424-436

Approximation-1:

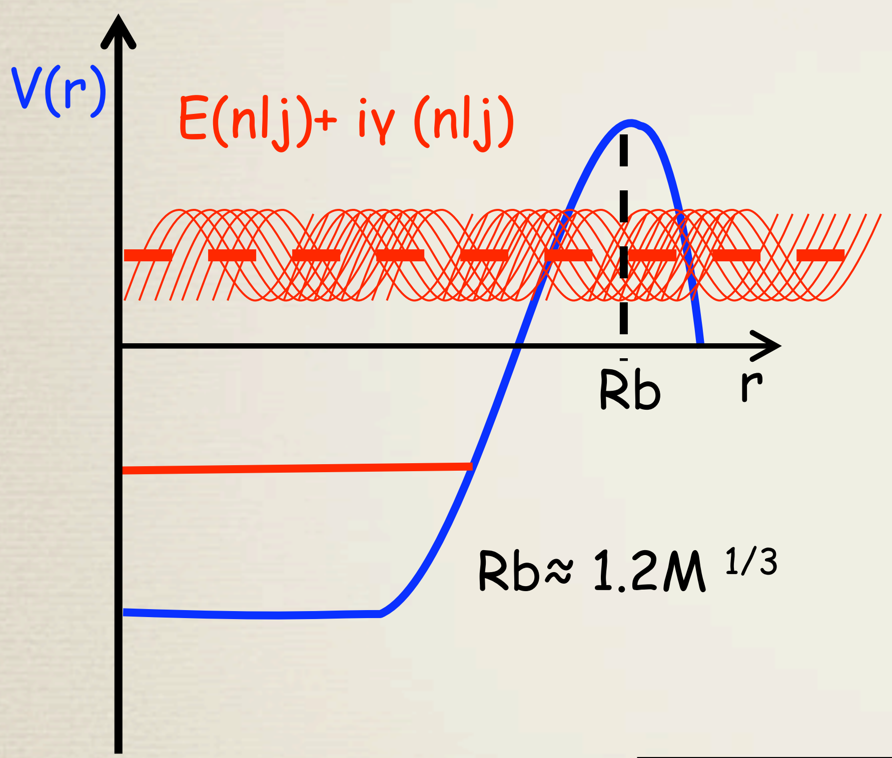
$\langle A ; i \mid H \mid A; f \rangle \sim \langle A ; i \mid Hsp \mid A; f \rangle$

Full transition operator can be approximated by a single-particle operator Hsp producing nucleon emission

$$\langle A, i \mid H \mid A, f \rangle = SPA(i ; r, nlj) \langle N(B), nlj \mid Hsp \mid N(U), Eklj \rangle$$

Approximation-2:

Gamow Theory: we want to describe nucleon emission with relative energy Ek → adjust depth of the single-particle potential to have a Gamow state at a complex energy E(nlj)+ iγ (nlj) so that



E(nlj) = Ek

γ (nlj; Ek): single-particle width for nucleon emission

Approximation-2bis: Semi-classical alternative

Simple barrier penetration model

γ (E) = hv(E) P(E)

P(Ek) =1/(1+ exp((EB−Ek)/ωb) — Parabolic barrier (Wong)

v (Ek)= √((V0+Ek)/ 2μRb²) — Bouncing freq. in infinite square well

$$\gamma(E) = \hbar \upsilon \, P(E) = \sqrt{\frac{E_k + V_0}{2\mu Rb^2}} \frac{\hbar}{1 + e^{\frac{Eb - Ek}{\omega_b}}}$$

Fermi Golden Rule:

$\gamma(nlj; Ek) = 2\pi \left| \langle N(B), nlj \mid Hsp \mid N(U), Eklj \rangle \right|^2 \rho(Ek)$

RESULTS

Decay width (nlj):

Γ (i; r ; lj ; Ek) = 2π $|\langle$ A ; i |H|A; f $\rangle|^2$ ρ (Ek) =
2π |SPA(i ; r, nlj)$|^2$ $|\langle$ N (B), nlj| Hsp |N (U), Eklj $\rangle|^2$ ρ(Ek)

→ Γ (i; r ; lj ; Ek) = |SPA(i ; r, nlj)$|^2$ γ (nlj; Ek)

Total width (r)

Γ (i; r ; Ek) = Σ_{lj} |SPA(i ; r, nlj)$|^2$ γ (nlj ; Ek)

Natural width of decaying state (only particle)

Γ_T (i) = Σ_r Γ (i; r ; Ek)

Branching ratio

b(i,r)= Γ (i; r ; Ek) / Γ_T (i)

Activity of nucleon emission

I(i,r) =I_β (i,r) b(i,r)

Three basic ingredients

1. Beta decay strenght
2. Spectroscopic amplitudes

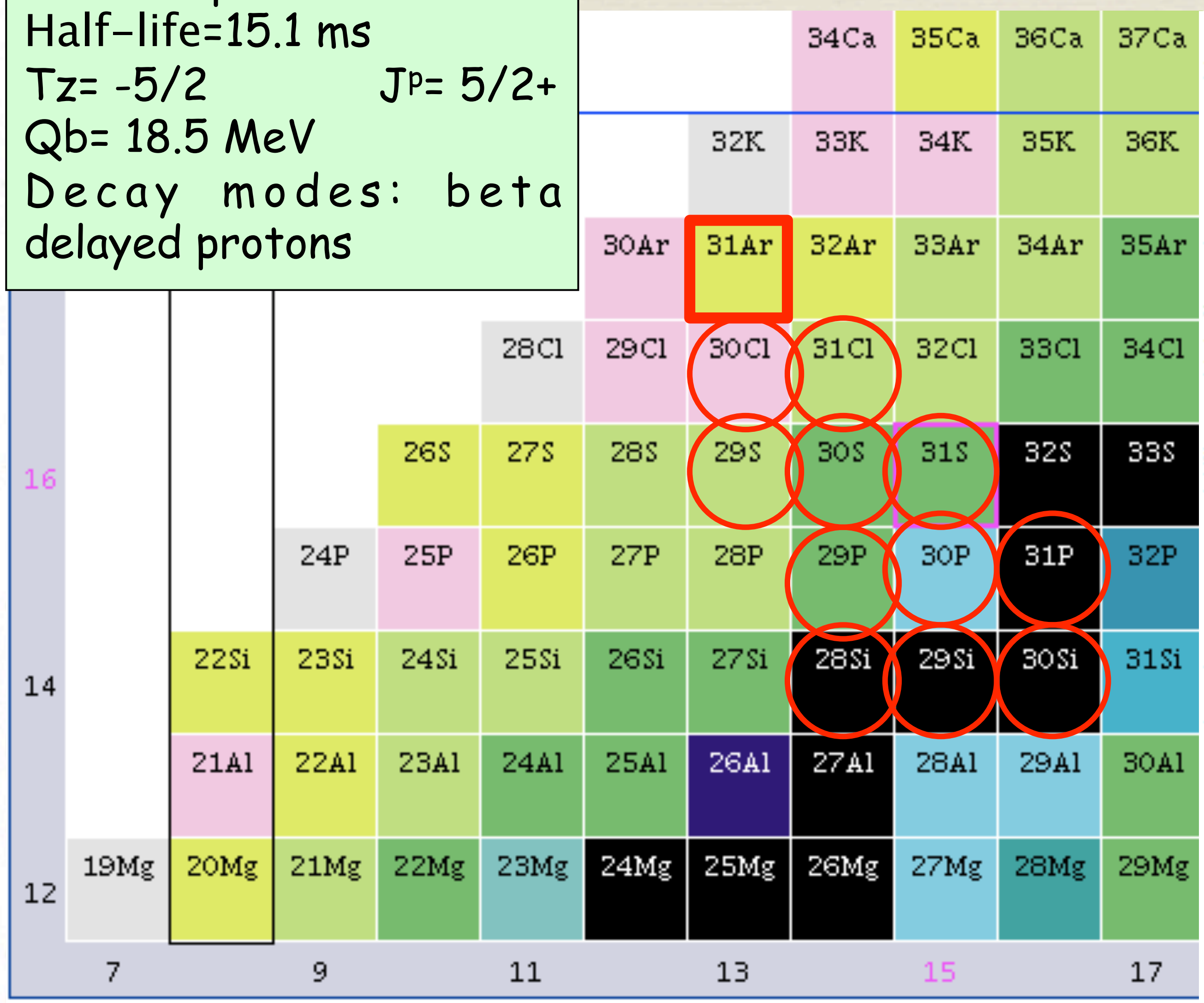
} Shell model calculation

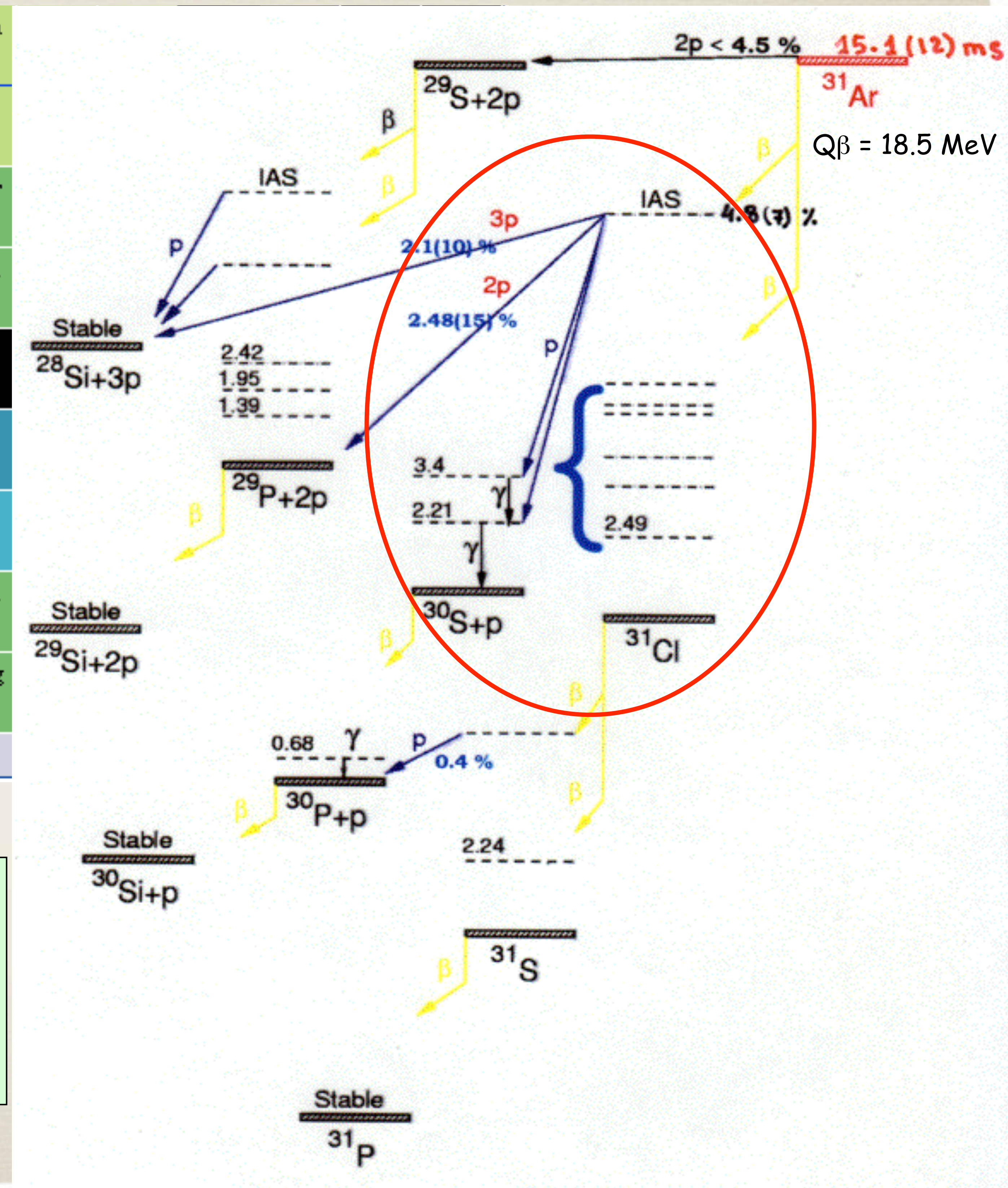
3. Single particle widths

{ Gamow state calculation~ Woods-Saxon (a=0.65fm, r=1.27fm), select depth V0 to reproduce Ek

Example: The case of beta delayed particle emission from 31Ar (Z=18, N=13)

31Ar: drip line nucleus
Half-life=15.1 ms
Tz= -5/2 Jp= 5/2+
Qb= 18.5 MeV
Decay modes: beta delayed protons

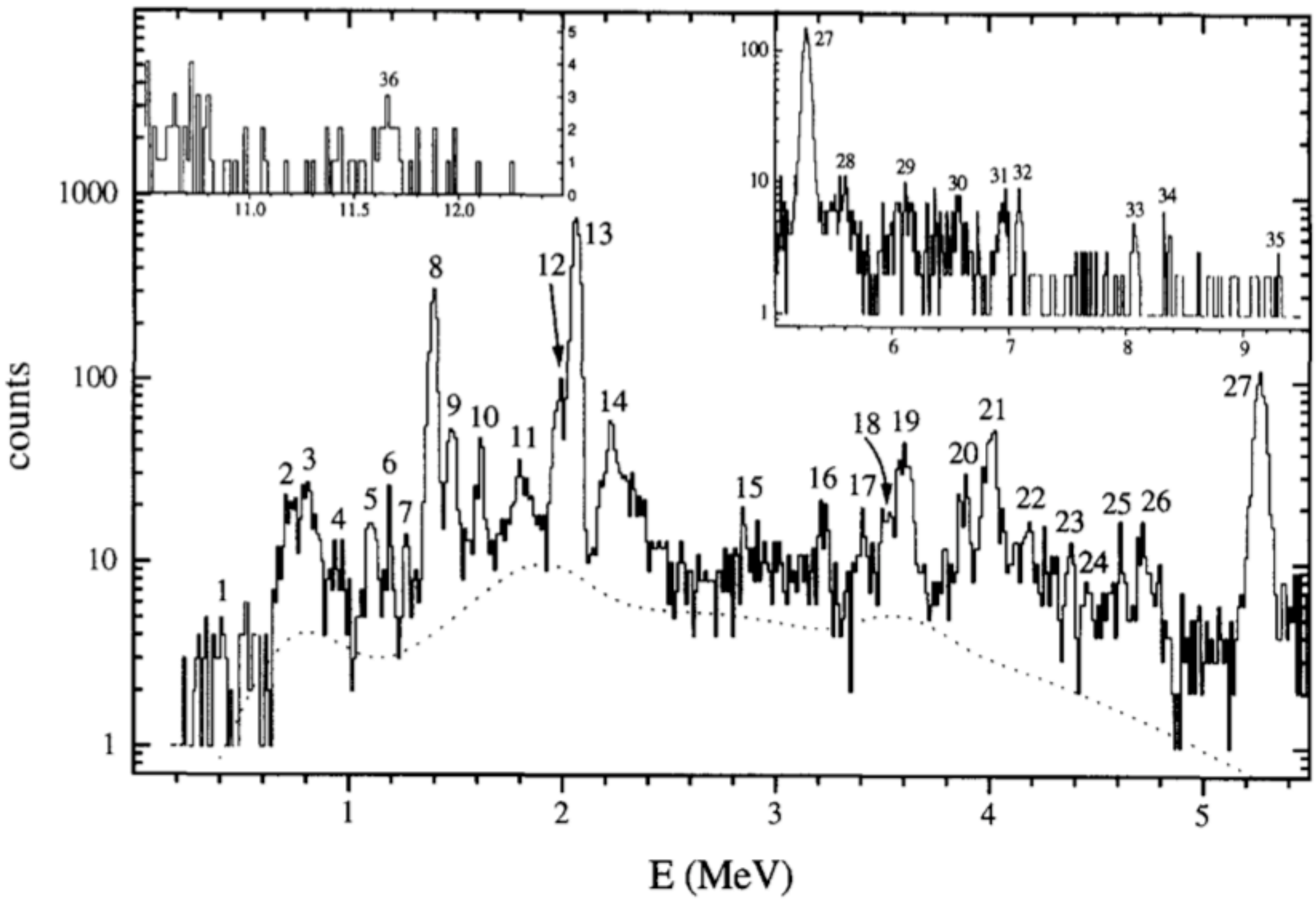




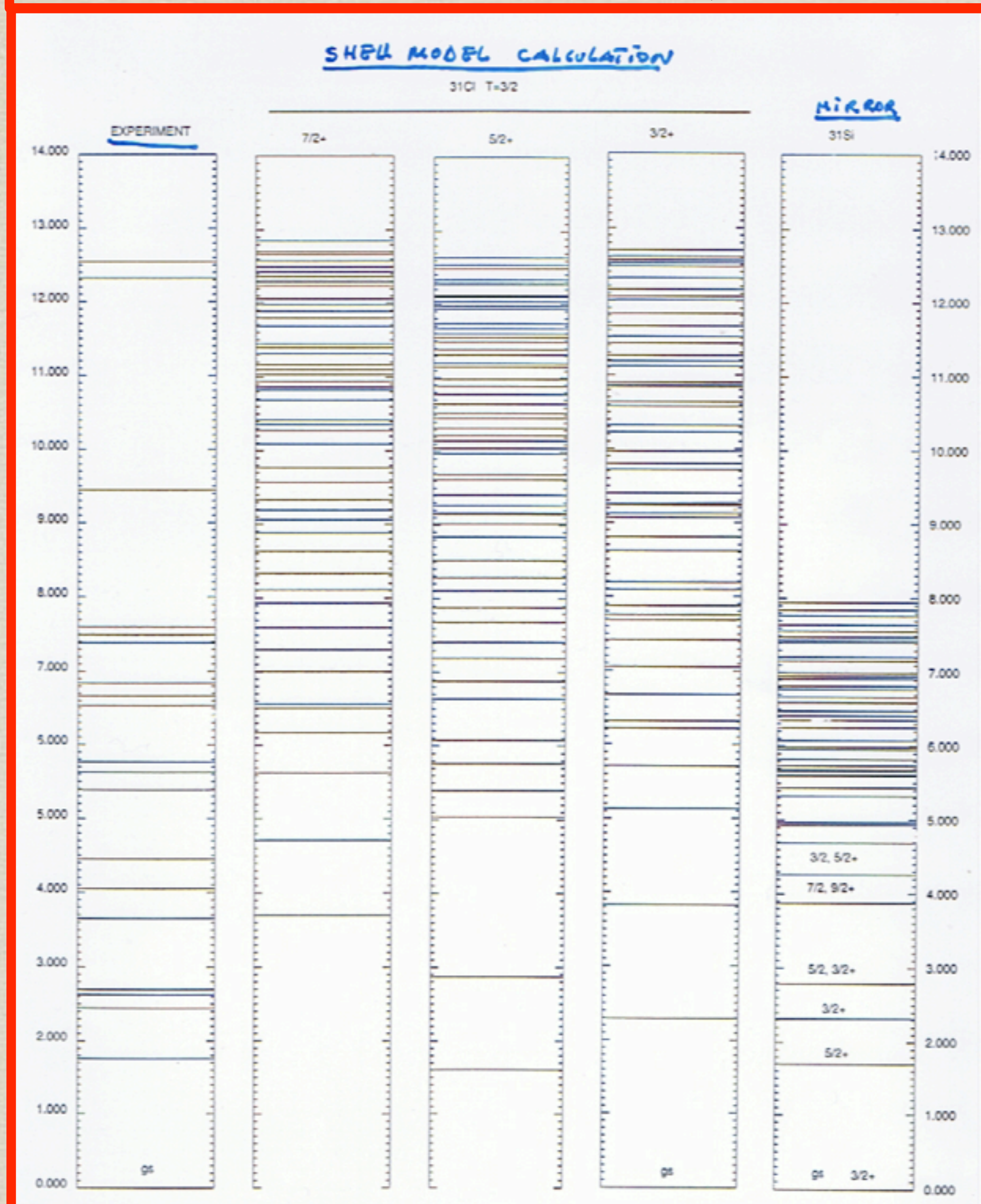
What is interesting:
High Qb: access to many levels of 31Cl; determine Jpi, widths, etc
P-emitter: bp, b2p, 3p...
B-2p emitter: Direct vs sequencial p-decay

EXPERIMENTAL PROTON SPECTRUM

L. Axelsson et al. / Nuclear Physics A 634 (1998) 475–496



SHELL MODEL CALCULATIONS (USD, Brown & Wildenthal)



430 I. Martel et al. / Nuclear Physics A 694 (2001) 424–436

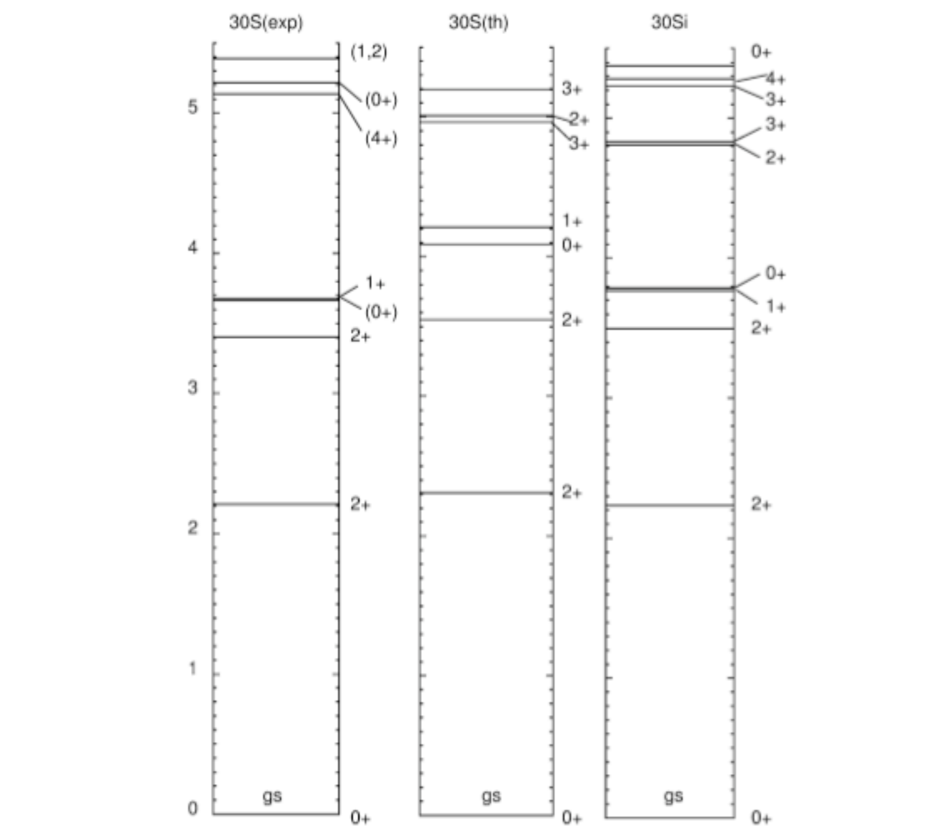
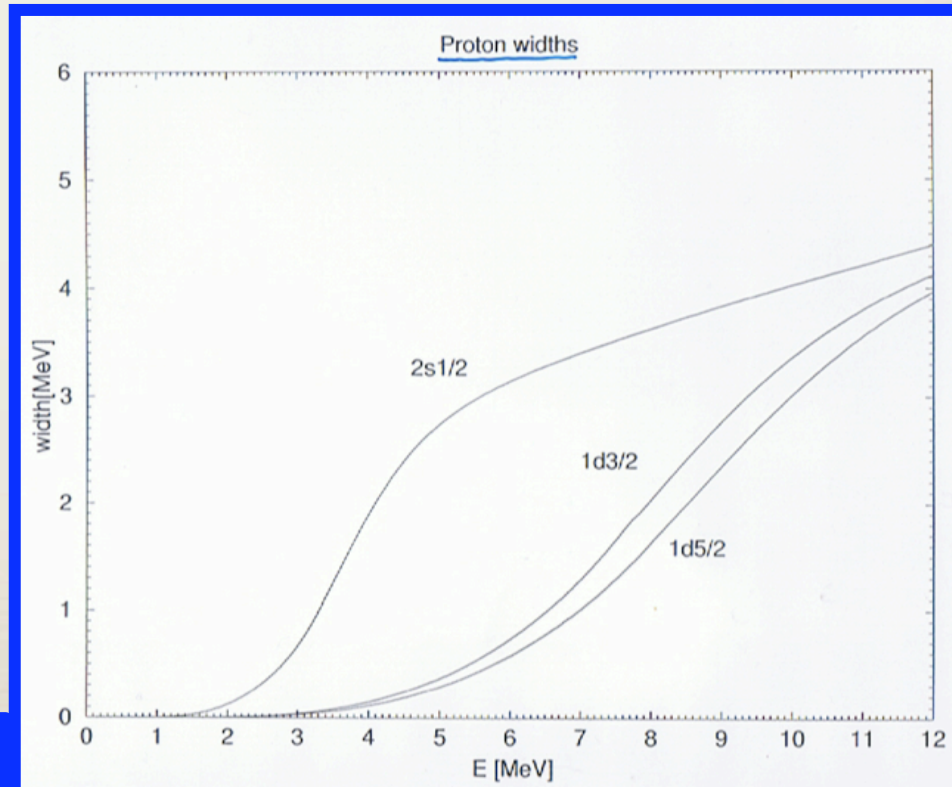
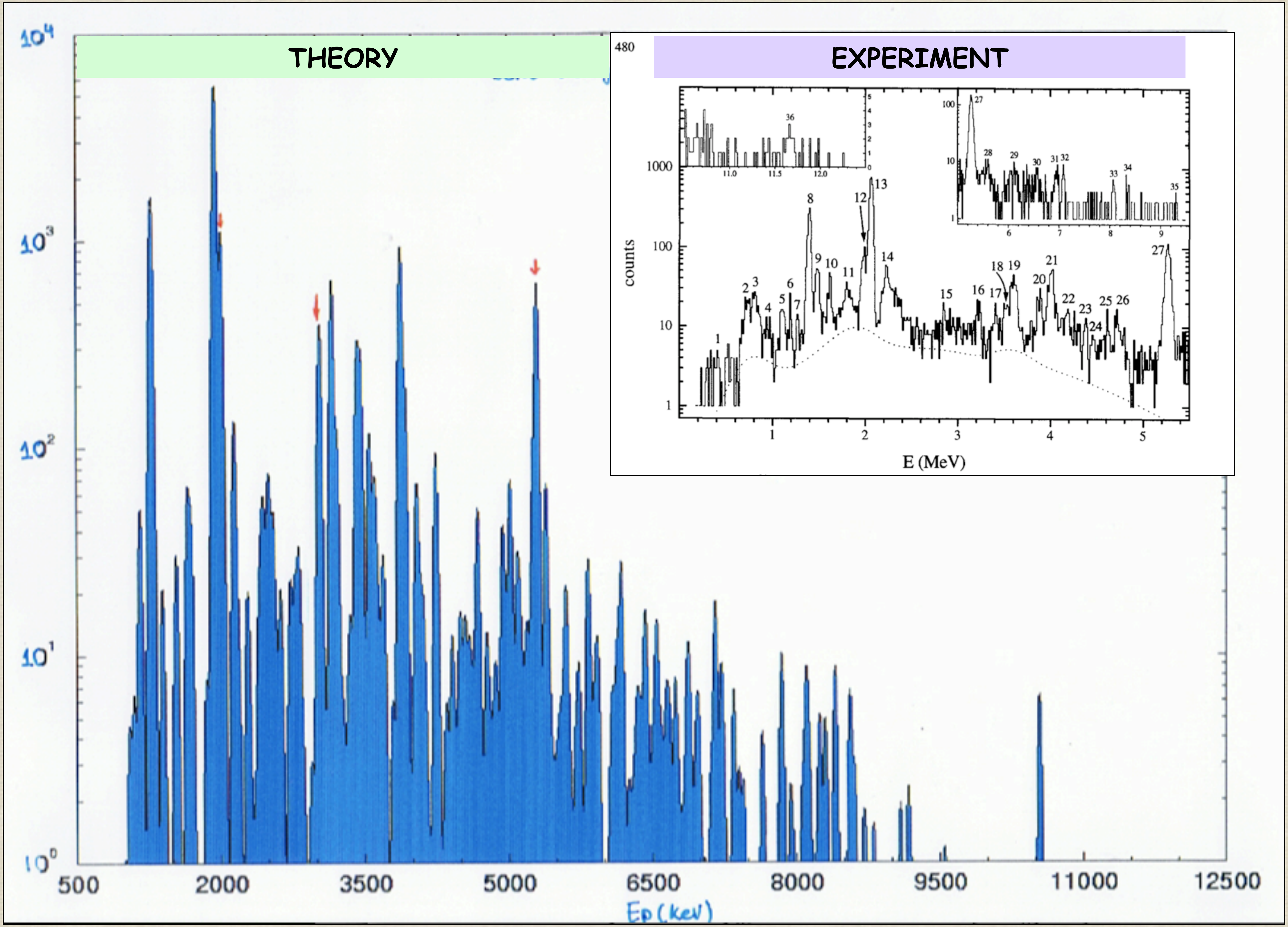


Fig. 2. Shell-model calculations for $T = 1$ levels in ^{30}S, the experimental values and the levels of the mirror nucleus ^{30}Si. Energy levels are given in MeV.



GAMOW STATE CALCULATIONS


THEORY
EXPERIMENT
480
counts
E (MeV)
Ep (keV)
500
2000
3500
5000
6500
8000
9500
11000
12500

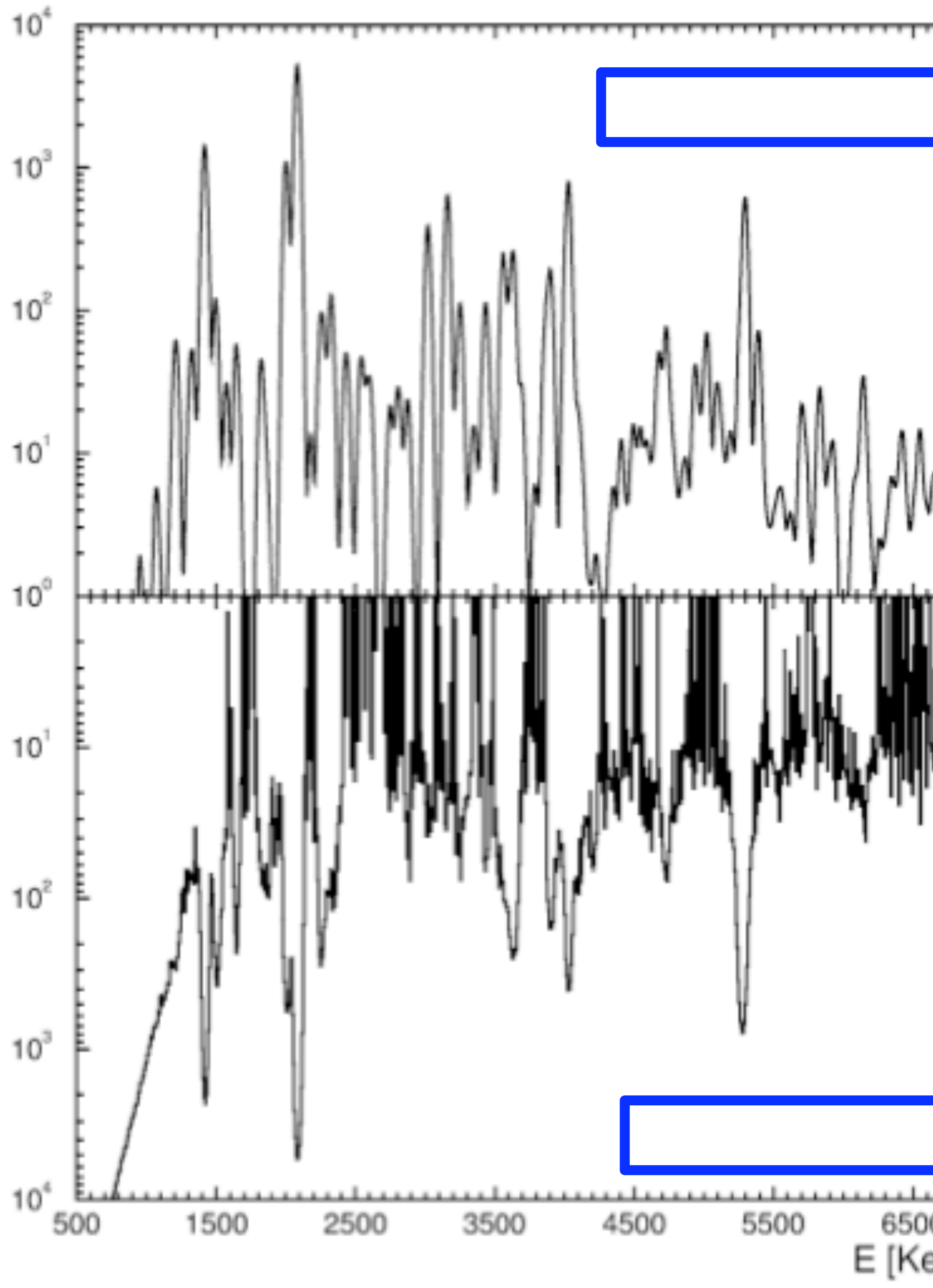


Fig. 3. Experimental energy spectrum of the beta-del
spectrum (up) after adjustment of level energies. Th
of 12 keV width.

Table 1
Results of the analysis of the beta-delayed proton energy spectrum

J^{π}_{Cl}	E_{Cl}	B(GT) (th)	J^{π}_{S}	E_p (exp)	I_p (th)	I_p (exp)	Γ (th)
$5/2^+_1$	1753	0.053	0^+_1	1416(2)	27.1	34.0(3)	0.02
$3/2^+_2$	2443	0.243	0^+_1	2084(2)	100	100.0(6)	0.4
$5/2^+_2$	2618	0.004	0^+_1	2253(2)	1.6	4.0(3)	0.8
$7/2^+_1$	3752	0.004	2^+_1	1211(4)	1.1	1.7(5)	0.11
$3/2^+_3$	4045	0.028	0^+	3634(3)	4.4	6.1(8)	9.8
			2^+_1	1504(2)	2.3	6.2(2)	5.1
$7/2^+_2$	4905	0.014	2^+_1	2327(4)	2.4	5.1(4)	1.7
$5/2^+_3$	5390	0.011	2^+_2	1643(2)	1.1	2.88(14)	3.4
$7/2^+_3$	5621	0.057	2^+_1	3020(3)	7.4	1.08(14)	3.1
$3/2^+_4$	5658	0.010	1^+_1	1643(2)	0.8	2.88(14)	3.8
$5/2^+_4$	5767	0.360	0^+_1	5276(5)	11.5	17.6(3)	7.4
			2^+_1	3153(4)	12.2	0.44(10)	7.8
			2^+_2	2008(2)	20.9	10.0(2)	13.4
$7/2^+_4$	6047	0.022	2^+_1	3432(3)	2.1	0.89(11)	9.8
$5/2^+_5$	6180	0.047	2^+_1	3561(11)	4.5	3.6(8)	30.8
$7/2^+_5$	6533	0.044	2^+_1	3902(3)	3.4	2.22(14)	11.3
$3/2^+_5$	6640	0.023	0^+_1	6145(7)	0.5	0.51(12)	5.4
$7/2^+_6$	6665	0.186	2^+_1	4030(3)	14.7	7.0(2)	4.6
			2^+_2	2881(3)	0.4	0.99(13)	0.13
$7/2^+_7$	7050	0.050	2^+_2	3249(4)	1.9	1.17(15)	2.6
			3^+_2	1300(13)	0.9	0.7(11)	1.3

LEVEL SCHEME

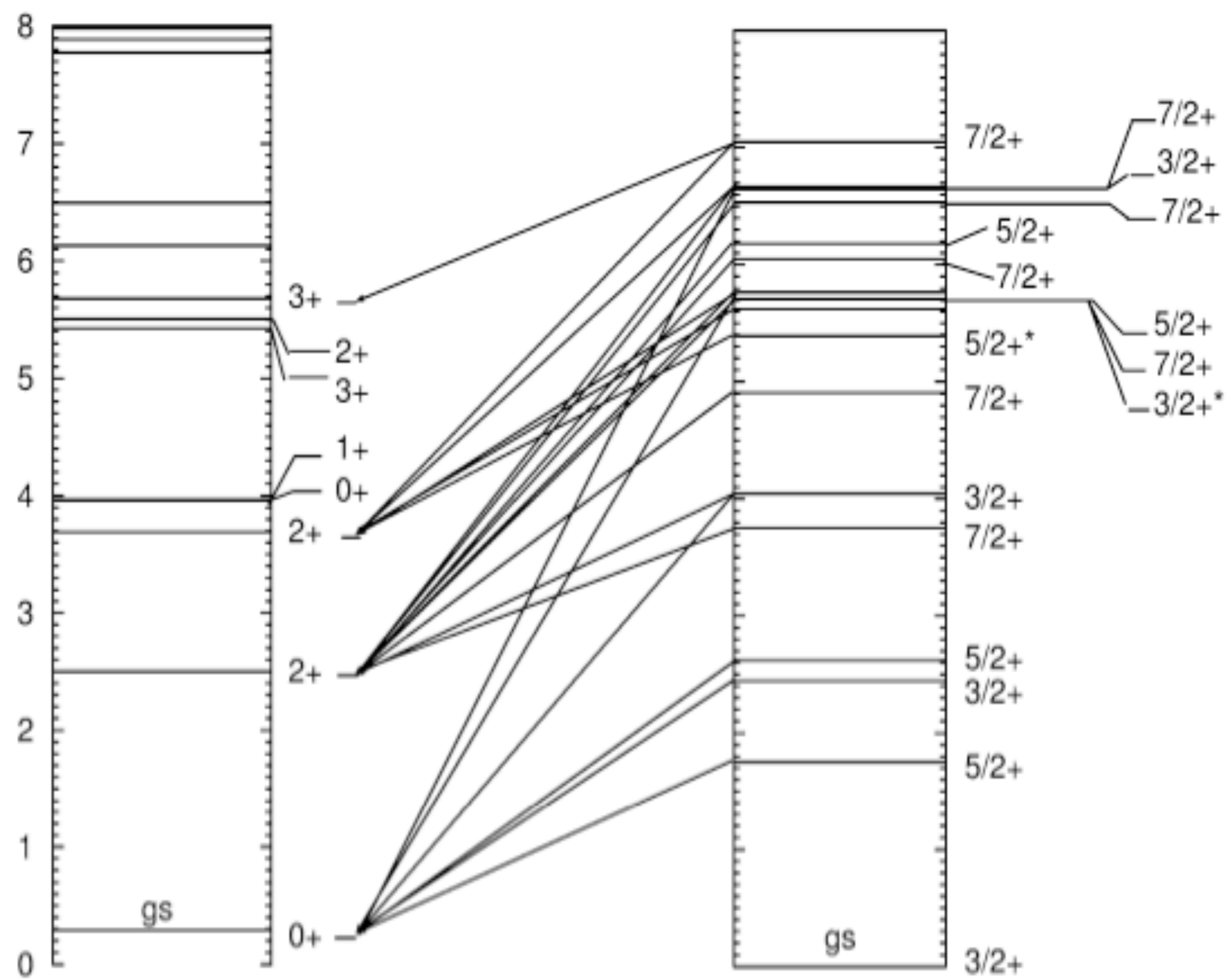


Fig. 4. The proposed level scheme and decay mechanism for ^{31}Cl. Energy levels are given in MeV, relative to the ground state of ^{31}Cl. (*) means ambiguous assignment; see text for details.

PROTON TRANSITIONs AND BRANCHING RATIOS

^{30}S STATES

n_s	$E(KeV)$	$J\pi$
1	0	0+
2	2210.6(5)	2+
3	3402.6(5)	2+
4	3666.3(13)	(0+)
5	3676(3)	1+
6	5136(2)	(4+)
7	5217.4(7)	
8	5389(2)	1,2
9	5842(4)	
10	5945(3)	3,4
11	6064(3)	1-
12	6202(3)	
13	6338.6(14)	
14	6541(4)	0+
15	6643(3)	2,3
16	6762(4)	
17	6855(4)	
18	6927(4)	>3
19	7078(7)	3,4
20	7123(10)	
21	7237(5)	1,2
22	7295(14)	
23	7352(8)	>2(1)
24	7485(4)	1,2
25	7598(4)	
26	7693(4)	
27	7924(5)	

n_{Cl}	$E_{Cl}(KeV)$	n_S	n_p	$I_\beta(\%)$	B(F)+B(GT)
1	0	1	-	23(8)	0.08(3)
2	1754(5)	1	4	9.0(9)	0.052(6)
3	2444(2)	1	11	26(3)	0.19(2)
4	2619(2)	1	12	1.05(13)	0.0080(11)
5	2695(4)	1	13	1.3(2)	0.011(2)
6	3649(4)	1	17	0.31(5)	0.0033(6)
7	4052(3)	1	20	3.2(4)	0.040(5)
		2	5		
8	4455(3)	1	23	1.8(2)	0.027(3)
9	5390(3)	3	6	0.76(9)	0.016(2)
10	5625(3)	2	15	0.50(7)	0.011(2)
		3	8		
11	5764(3)	1	28	7.4(8)	0.18(2)
		2	16		
		3	10		
12	6534(3)	1	31	0.71(9)	0.024(3)
		2	22		
13	6668(2)	1	32	1.6(2)	0.055(8)
		3	14		
		6	2		
		7	1		
14	6841(5)	7	3	0.18(3)	0.0070(14)
15	7386(4)	2	27	2.2(3)	0.11(2)
		3	19		
		7	7		
16	7496(4)	1	35	0.53(8)	0.027(4)
		4	18		
		7	9		
17	7602(10)	1	36	0.13(2)	0.0070(13)
18	9455(5)	1	39	0.21(6)	0.031(8)
		7	21		
19	12322(2)$^{(*)}$	1	41	4.25(30)	$3.2(^{+3.0}_{-0.3})$
		2	40		
		3	38		
		4	37		
		7	34		
		8	33		
		9	30		
		12	29		
		25	26		
		21	25		
		27	24		
20	12547(30)	1	42	0.0090(12)	0.013(2)

BETA STRENGHT

n_p	$E_p(KeV)$	$I_p(\%)$
1	1131(5)	2.7(16)
2	1211(4)	1.7(5)
3	·1300(13)	0.7(11)
4	1416(2)	34.0(3)
5	1504(2)	6.2(2)
6	1643(2)	2.88(14)
7	1819(3)	3.0(4)
8	1870(3)	0.8(2)
9	1923(3)	0.44(14)
10	2008(2)	10.0(2)
11	2084(2)	100.0(6)
12	2253(2)	4.0(3)
13	2327(4)	5.1(4)
14	2881(3)	0.99(13)
15	3020(3)	1.08(14)
16	3153(4)	0.44(10)
17	3249(4)	1.27(15)
18	3432(4)	0.89(11)
19	3561(3)	3.6(8)
20	3634(11)	6.1(8)
21	3806(3)	0.53(13)
22	3902(4)	2.22(14)
23	4030(3)	7.0(2)
24	4200(3)	1.09(18)
25	4289(4)	0.31(8)
26	4389(5)	0.59(11)
27	4730(5)	1.68(18)
28	5276(5)	17.6(3)
29	5632(6)	0.37(9)
30	5952(7)	0.19(6)
31	6049(9)	0.51(12)
32	6145(7)	0.51(12)
33	6386(7)	0.26(5)
34	6540(8)	0.84(11)
35	6950(9)	0.70(9)
36	7074(9)	0.49(7)
37	8092(14)	0.25(4)
38	8347(15)	0.51(6)
39	8860(19)	0.22(19)
40	9493(20)	0.30(4)
41	11654(28)	0.27(4)
42	11858(29)	0.034(3)

SUMMARY

We have revised the physics concepts behind the beta delayed particle emission process:

- Basic ides about the exotic decay process

- Exotic decays are an important source of spectroscopic information: level energies, spins, B(F) and B(GT) values, etc

- Technical aspects to measure these decay modes

- Status of beta delayed nucleon emission

- Basic ideas for beta decay and isospin

- Simple models for particle emission (Gamow states, R-Matrix,…)

- Decay rates obtained using very simple models describe well exotic radioactivity

THANKS FOR YOUR ATTENTION…