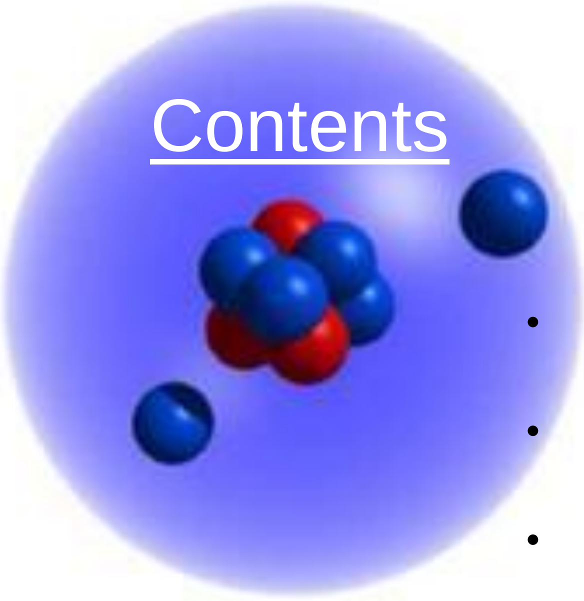
Nuclear Reactions:
Lecture on two-body description

Ismail Boztosun

AKDENİZ ÜNİVERSİTESİ 1982

Feb 2013, Warsaw, boztosun@akdeniz.edu.tr

Contents



- Fundemantels: Types of nuclear reactions
- Observables
- Compound vs Direct reactions
- Collision theory-elastic scattering
  - Lab vs CM koordinate systems
- Effective potential
- Born and DWBA
  - examples
- Optical model
- Coupled-channels model-Inelastic scattering

Suggested books

G.R. Satchler,
Introduction to Nuclear Reactions, Oxford Uni. Press.

I. Thompson & F. Nunes,
Nuclear Reaction for Astrophysics, Cambridge Uni. Press.

C. Bertulani & P. Danielewicz,
Introduction to Nuclear Reactions, Taylor&Francis

L.S. Rodberg & R.M. Thaler,
Introduction to Quantum Scattering Theory, Academic

Advanced Level;
G.R. Satchler, Direct Nuclear Reactions, Oxford Uni. Press
N. Austern, Direct Nuclear Reactions Theories, Wiley
N. K. Glendenning, Direct Nuclear Reactions, World Scientific

Types of Nuclear Reactions

Scattering vs Reaction

1- Elastic Scattering (always present) , No energy change X(a,a)X

$a+A \rightarrow a+A$, $Q=0$, $^{24}Mg(^{12}C,^{12}C)^{24}Mg$

2- Inelastic Scattering , Energy change X(a,a*)X*

$a+A \rightarrow a^*+A^*$, $Q<0$, $^{24}Mg(^{12}C,^{12}C^*)^{24}Mg^*$

3- Inelastic Reaction, Energy and nucleon change

$a+A \rightarrow b+B$, $^{14}N(^{4}He,p)^{17}O$

4- Radiative Capture

$a+A \rightarrow B+\text{gamma}$, $^{12}C(^{4}He,\text{gamma})^{16}O$

5- Photo Reaction

$\text{gamma}+A \rightarrow B+b$, $^{16}O(\text{gamma}, ^{4}He)^{12}C$

6- Transfer Reaction

X(d,p) or X(p,d), $^{12}C(^{6}He, ^{4}He)^{14}C$

$$Q = (m_{initial} - m_{final})c^2$$

Observables

Experiment:

- Angular distribution: E fixed, θ variable
- Excitation function: θ fixed, E variable

Theory:

- "Direct" problem: determine cross-section from the potential
- "Inverse" problem : determine the potential V from cross-section

Coulomb field (potential) ⇒ Coulomb scattering
- elastic ⇒ Rutherford scattering
- inelastic ⇒ Coulomb excitation

Nuclear field (potential)
- near nuclear surface ⇒ shape-elastic scattering (potential scattering)
- inside nucleus ⇒ resonant scattering (compound-elastic scattering)

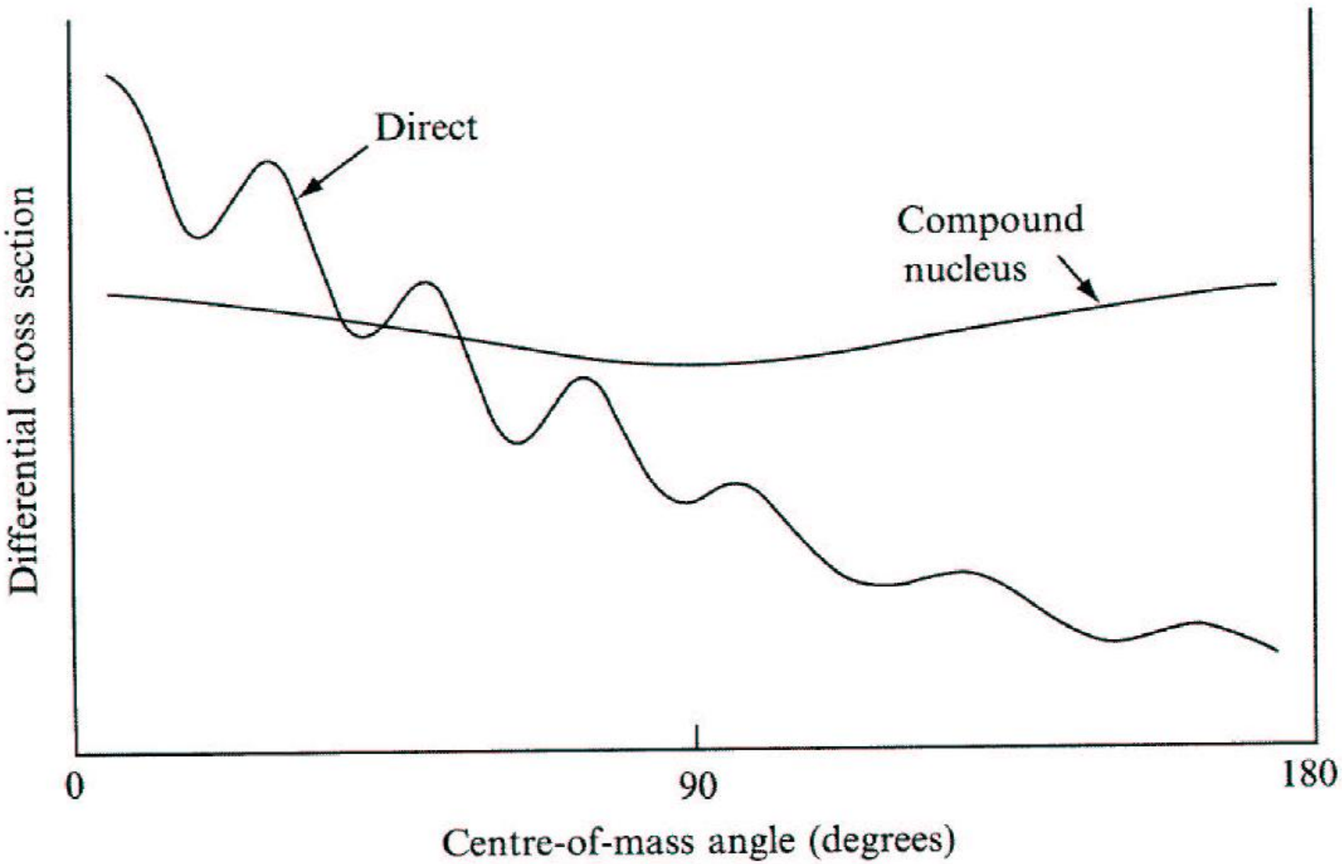
Nuclear Reactions:

Compound **Direct**

- Sma
- Low
- Two
  entra
  a+A
  C →*
- Time
- Isotr
  secti

pheral)

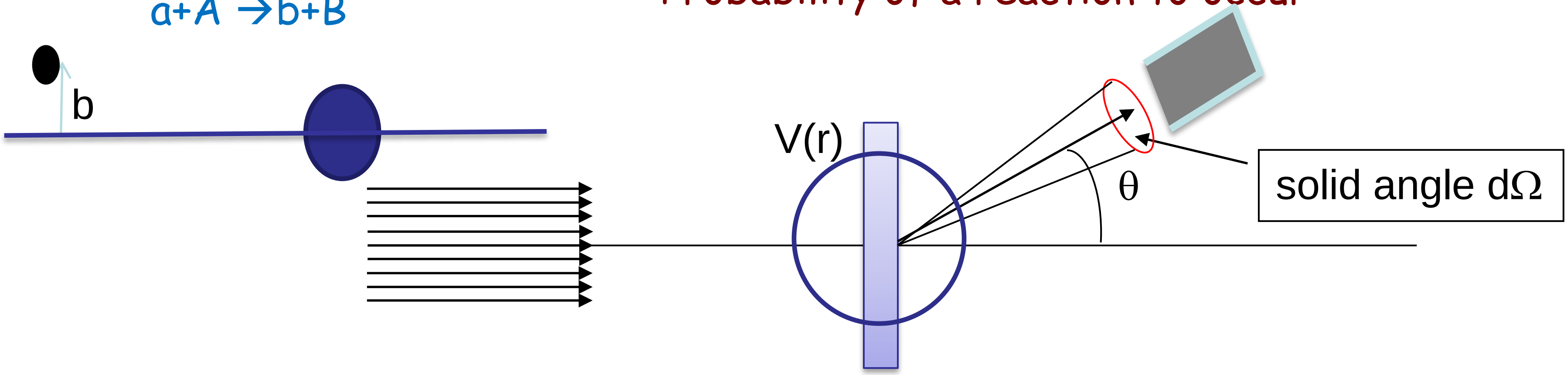
cillatory



Cross-section:

$a + A \rightarrow b + B$

Probability of a reaction to occur



I_a = current of incident particles a $\Rightarrow$ no. particles / unit time
N = no. target nuclei / unit area
R_b = no. detected particles b / unit time ("rate")

Proportionality relation: $R_b \sim I_a N$

<u>*Define:*</u>

cross section σ

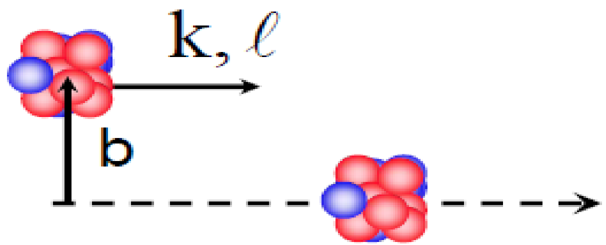
$$\sigma = \frac{R_b}{I_a N}$$

cross section has dimension of AREA
unit for cross section σ:

barn = 10^{-24} cm^2 = 100 fm^2

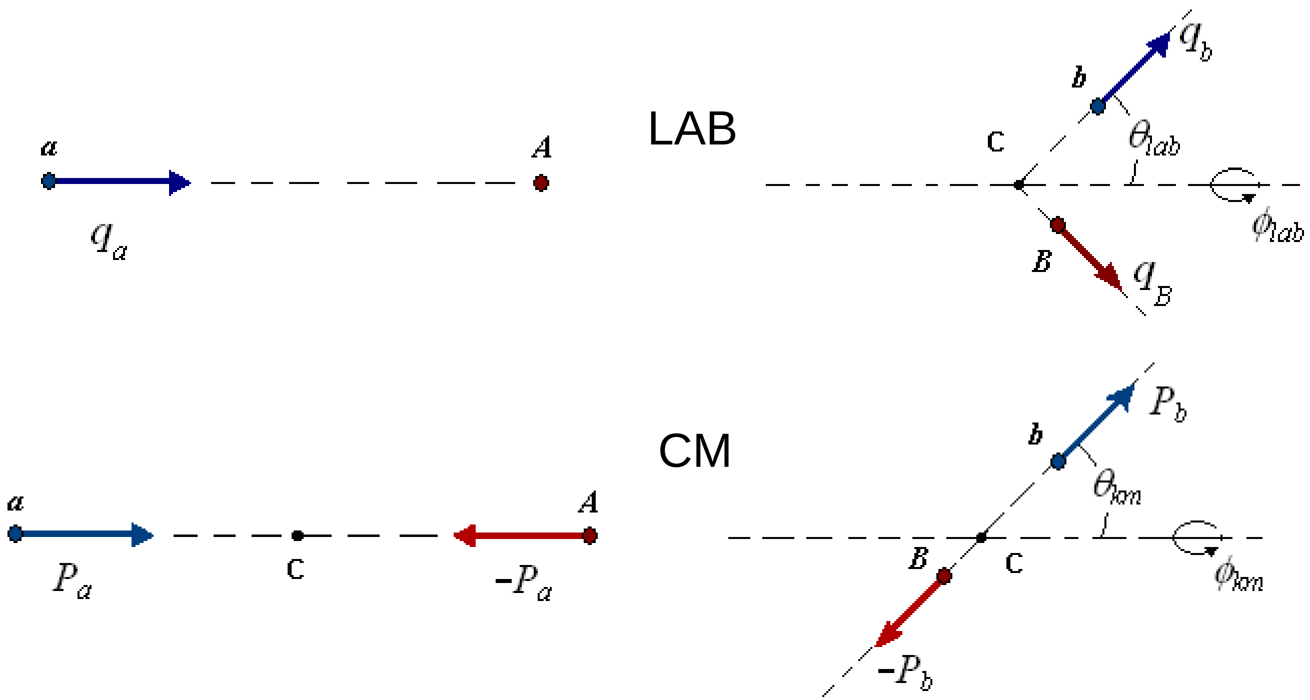
Collision theory: elastic scattering

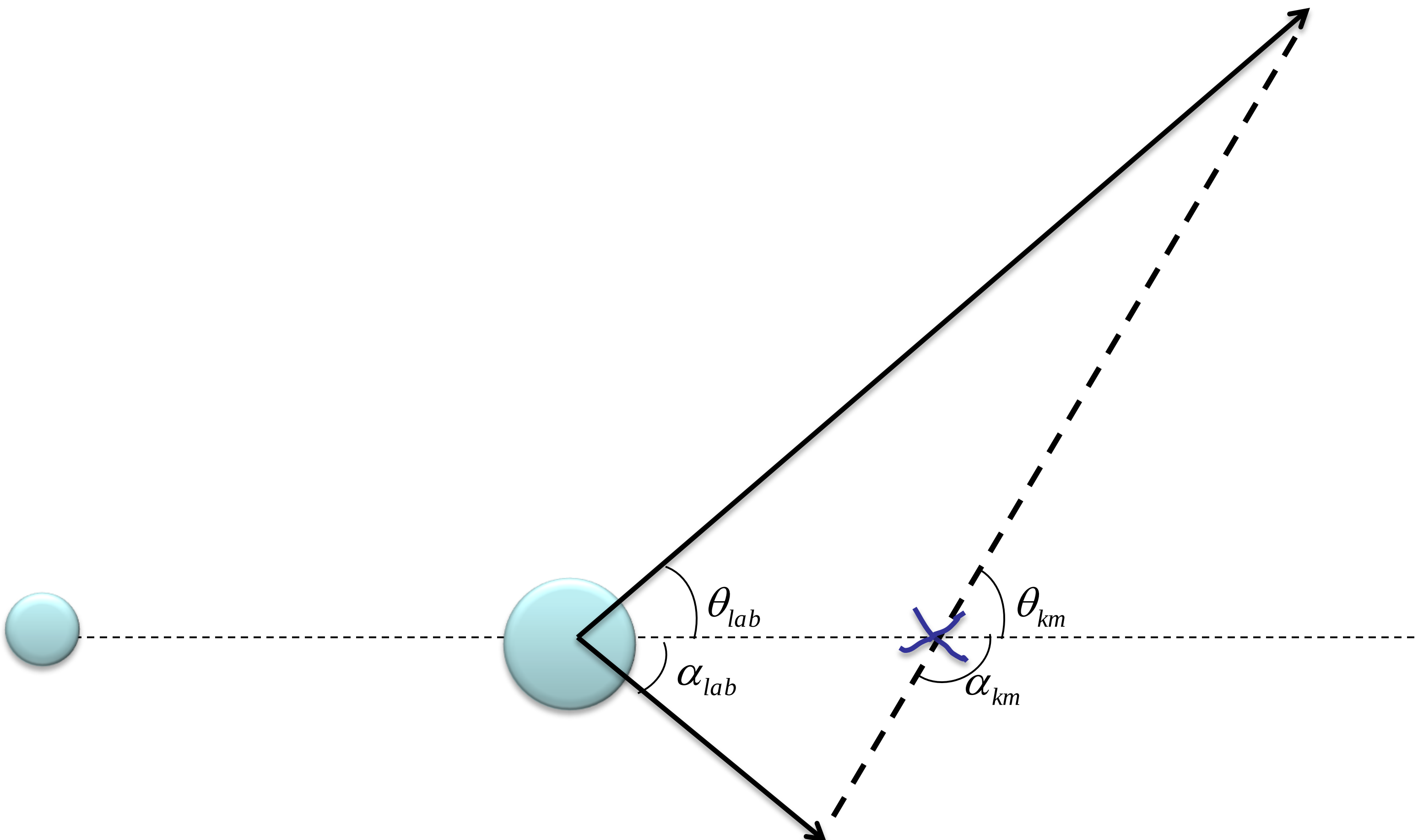
b=impact parameter



Before collision

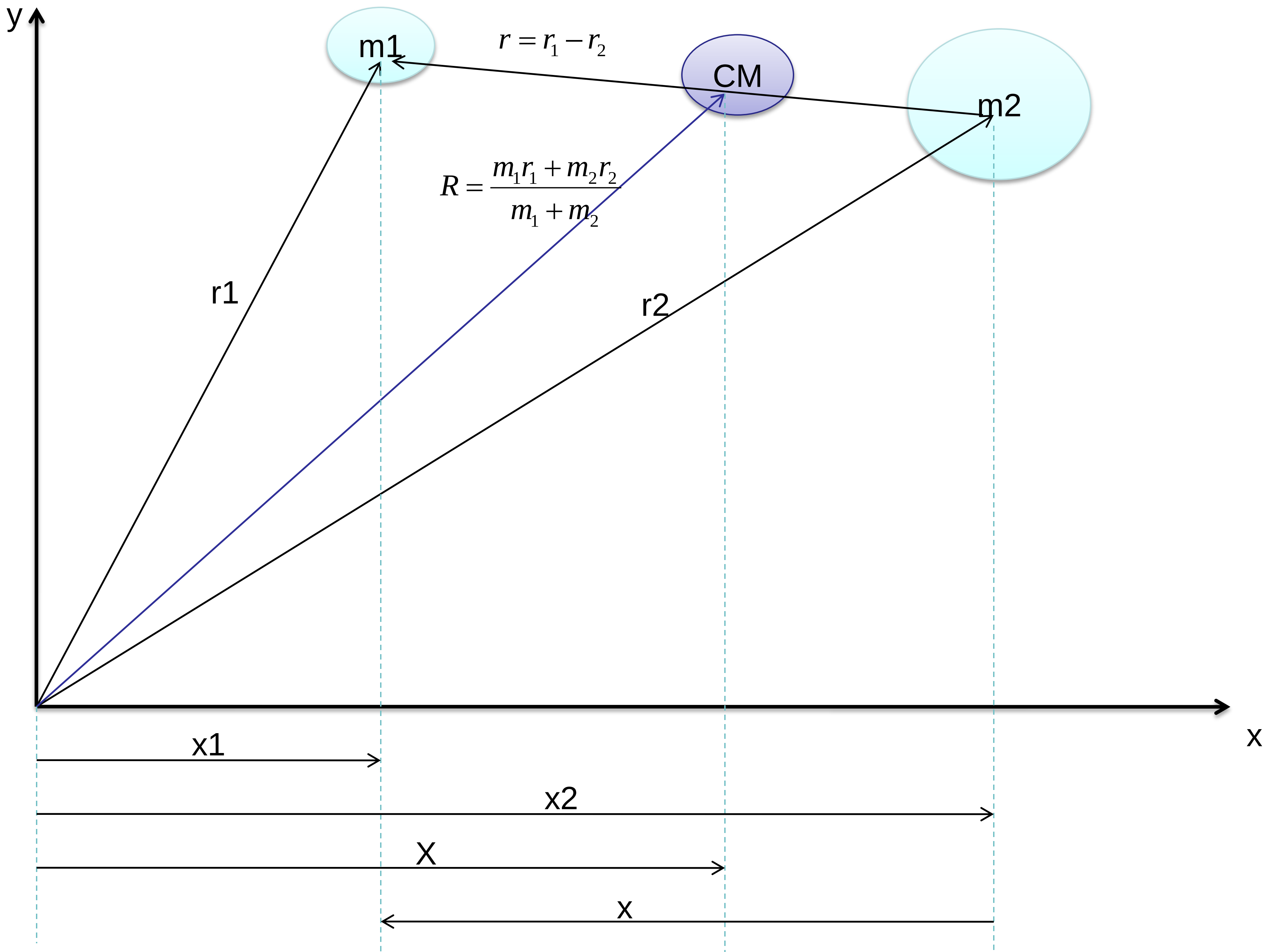
After collision





$$\tan(\theta_{lab}) = \frac{\sin(\theta_{cm})}{\cos(\theta_{cm}) + v_M / v_{cm}}$$

$$E_{CM} = \frac{m_2}{m_1 + m_2} E_{lab}$$


y
m1
$r = r_1 - r_2$
CM
m2
$R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$
r1
r2
x
x1
x2
X
x

$$\left(-\frac{\hbar^2}{2m_1}\nabla_1^2-\frac{\hbar^2}{2m_2}\nabla_2^2+V(r_1,r_2)\right)\psi=E\psi$$

$$\nabla_i\psi=\frac{\partial\psi}{\partial x_i}+\frac{\partial\psi}{\partial y_i}+\frac{\partial\psi}{\partial z_i}$$

$$\frac{\partial\psi}{\partial x_1}=\frac{\partial\psi}{\partial X}\frac{\partial X}{\partial x_1}+\frac{\partial\psi}{\partial x}\frac{\partial x}{\partial x_1}$$

$$\frac{\partial X}{\partial x_1}=\frac{m_1}{m_1+m_2}$$

$$\frac{\partial x}{\partial x_1}=1$$

$$\frac{\partial\psi}{\partial x_1}=\frac{m_1}{m_1+m_2}\frac{\partial\psi}{\partial X}+\frac{\partial\psi}{\partial x}$$

$$\left(-\frac{\hbar^2}{2[m_1+m_2]}\nabla_R^2-\frac{\hbar^2}{2[m_1m_2/(m_1+m_2)]}\nabla_r^2+V(r)\right)\psi=E\psi$$

$$m=m_1m_2/(m_1+m_2)$$

$$M=m_1+m_2$$

$$-\frac{\hbar^2}{2M}\nabla_R^2\psi-\left(\frac{\hbar^2}{2m}\nabla_r^2+V(r)\right)\psi=E\psi$$

R

Center of Mass

$$-\frac{\hbar^2}{2M}\nabla_R^2\psi=\varepsilon_0\psi$$

r

Relative Motion

$$-\left(\frac{\hbar^2}{2m}\nabla_r^2+V(r)\right)\psi=\varepsilon\psi$$

Two body problem: Schrödinger Equation with effective potential

Two Body Problem

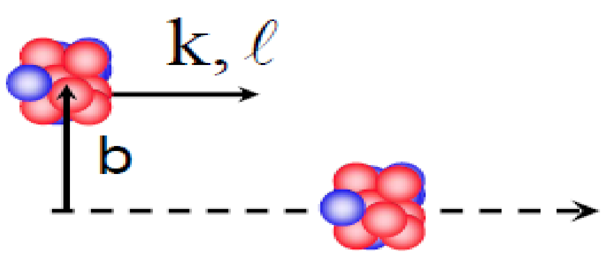
1. Move from LAB system to CM.
2. Seperate the CM motion
3. Find the reduced mass

$$m = \frac{m_1 m_2}{m_1 + m_2}$$

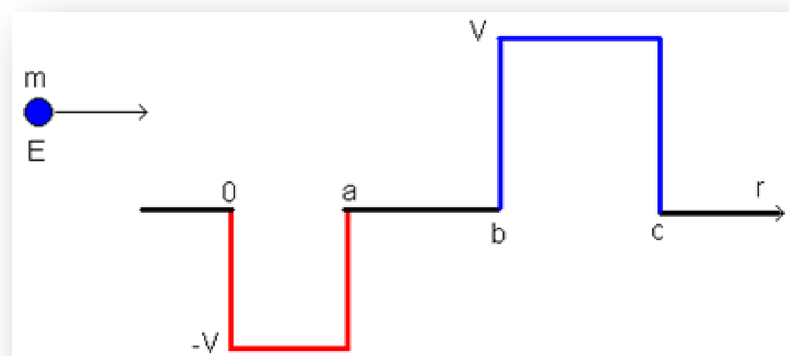
4. Define the total Central Potential V(r)
5. Solve the Sch. Eq. For this V(r).

"Reducing the interaction of projectile and target nuclei to interaction of the reduced mass with a potential between them."

b=impact parameter



=

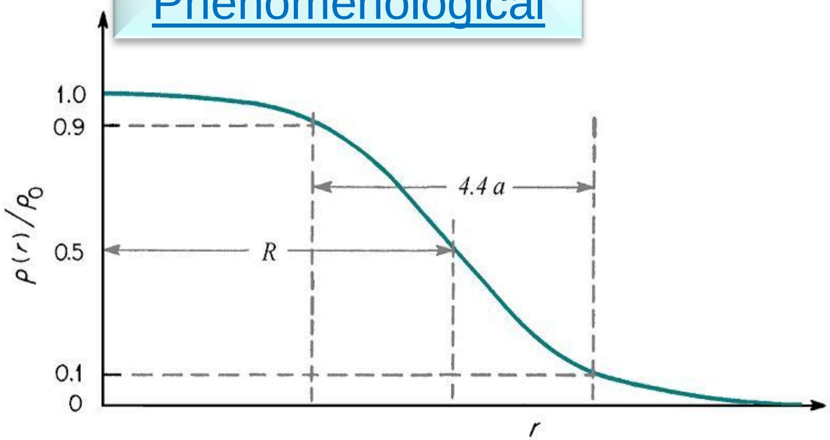


Effective Potential

$$V(r) = V_N(r) + \frac{Z_1 Z_2 e^2}{r} + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}$$

Nuclear Potentials

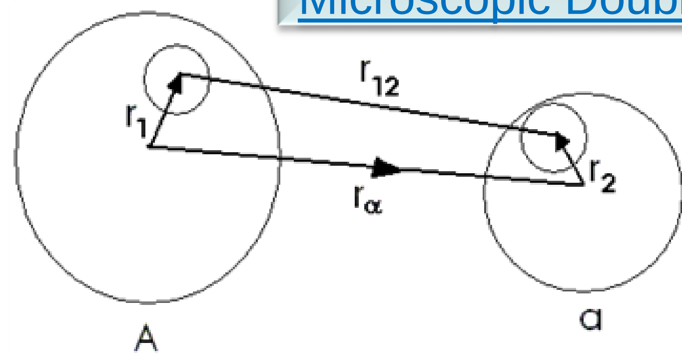
Phenomenological



$$V(r) = \frac{-V_0}{1+\exp\left[\frac{r-R}{a}\right]^n}$$

$$R = r_0 A^{1/3}$$

Microscopic Double Folding



$$U_F(\vec{R}) = \int d\vec{r}_1 \int d\vec{r}_2 \rho_1(\vec{r}_1)\rho_2(\vec{r}_2)\nu(\vec{R}-\vec{r}_1+\vec{r}_2)$$

$\rho_1(\vec{r}_1) \longrightarrow$ Density of A

$\rho_2(\vec{r}_2) \longrightarrow$ Density of a

$\nu(\vec{R}-\vec{r}_1+\vec{r}_2) \longrightarrow$ NN interaction +exchange term

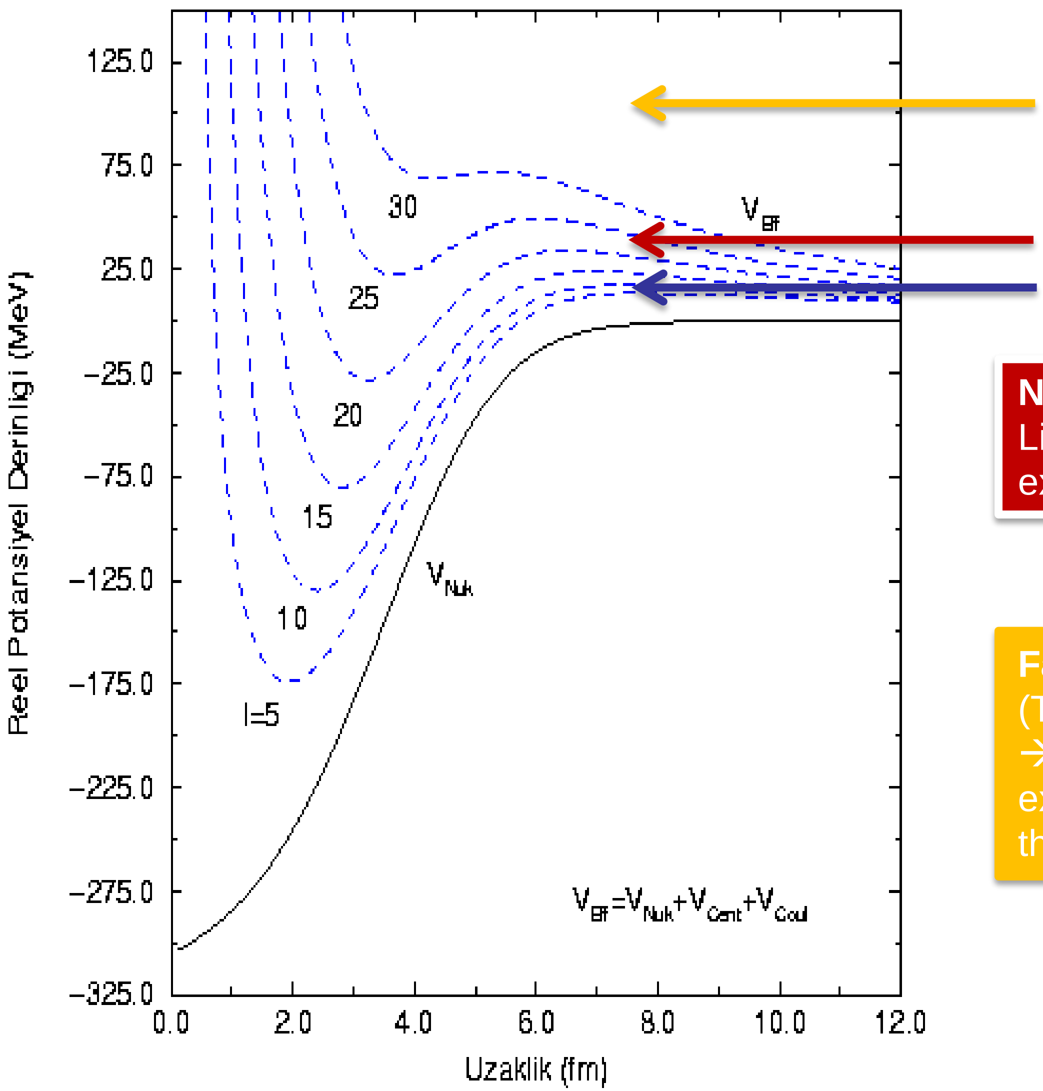
$$\nu(r) = 7999\frac{\exp(-4r)}{4r} - 2134\frac{\exp(-2.5r)}{2.5r} + J_{00}(E)\delta(r)$$

$$V(r) = V_N(r) + \frac{Z_1 Z_2 e^2}{r} + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}$$

Coulomb Barrier vs Energy

$R_B \sim A_1^{1/3} + A_2^{1/3}$

$V_B = 1.44\, Z_1 Z_2 / R_B$



Below the barrier
Very few ℓ values
R matrix

Nuclear Astrophysics

Near the barrier
Limited ℓ values→ partial wave expansion
ex: Optical, CC, CDCC, DWBA, etc.

Far above the barrier
(Too) many ℓ values
→ no partial wave expansion
ex: Eikonal, Glauber, semi-classic theories, etc.

Two body problem: Numerical Solution of Schrödinger Equation

Analytic solution of the Schrödinger equation is limited to few potentials: SW, HO

In general, there is no analytic solution → numerical approach

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}u_\ell(r) + (V(r) - E)u_\ell(r) = 0$$

$$u_\ell(r) \to F_\ell(kr,\eta)\cos\delta_\ell + G_\ell(kr,\eta)\sin\delta_\ell$$

Numerical solution : discretization N points, with mesh size h

- $u_\ell(0)=0$, $u_\ell(h)=1$ (or any constant)
- $u_\ell(2h)$ is determined numerically from $u_\ell(0)$ and $u_\ell(h)$ (Numerov algorithm)
- $u_\ell(3h)$,… $u_\ell(Nh)$
- for large r: matching to the asymptotic behaviour → phase shift

Bound states (E<0): same idea

Born and Distorted Wave Born Approximation

$$L_k(r)\psi(r) = U(r)\psi(r)$$ where $$L_k(r) = \nabla^2 + k^2$$

Multiplying by $L_k^{-1}(r)$ and integrating all over the space we get

$$\psi(r) = \phi_k(r) + \int U(r')\psi(r')L_k^{-1}(r)\delta(r'-r)dr'$$

$\phi_k(r) = e^{ikr}$ is the free particle solution (V=0). Using Green functions:

$$L_k^{-1}(r)\delta(r'-r) = G_k(r-r')$$

$$G_k^+(r-r') = -\frac{1}{4\pi}\frac{e^{ik|r-r'|}}{|r-r'|}$$

$$\psi_k^+(r) = \phi_k(r) - \frac{1}{4\pi}\int \frac{e^{ik|r-r'|}}{|r-r'|}U(r')\psi_k^+(r')dr'$$

$$\frac{1}{r-r'} \approx \frac{1}{r} \qquad k|r-r'| \approx kr - k.r'$$

$$\psi_k^+(r) = \phi_k(r) - \frac{e^{ikr}}{4\pi r}\int e^{-ik'.r'}U(r')\psi_k^+(r')dr'$$

Scattering Amplitude and Cross-section

$$f(\theta,\varphi) = -\frac{1}{4\pi}\int e^{-ik'.r'} U(r')\psi_k^+(r')dr'$$

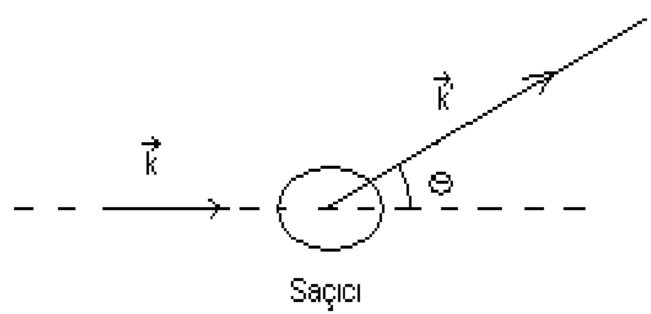
If we use plane wave for the $\psi_k^+(r)$ scattering amplitude in Born Approximation is:

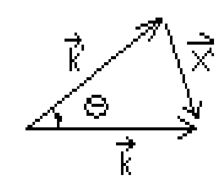
$$f_{BA}(\theta,\varphi) = -\frac{1}{4\pi}\int e^{-ik'.r'} U(r') e^{ik.r'} dr'$$

$$q = k - k'; \;\; f_{BA}(\theta,\varphi) = -\frac{1}{4\pi}\int U(r) e^{iqr} d^3r$$

For spherically symmetric potential $\int U(r) e^{iqr} d^3r = \frac{4\pi}{q}\int U(r) r \sin(qr) dr$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2, \quad \sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$





Distorted Wave Born Approximation

$$U = U_1 + U_2 \; such \quad that \; U_1 > U_2$$

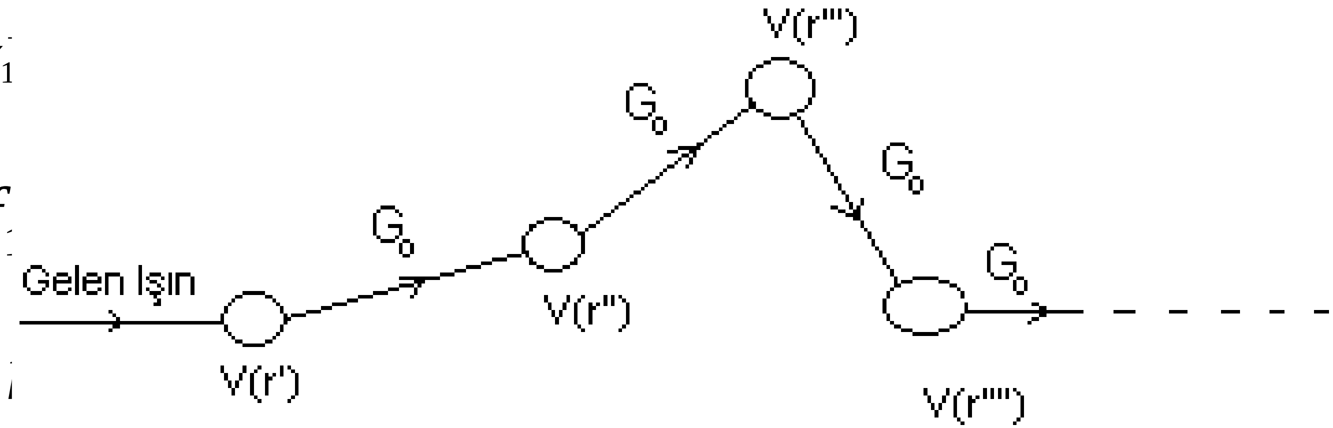
Incoming and outgoing waves

$$[\nabla^2 + k^2 - U_1(r)]\chi_1(k,r) = 0 \qquad \chi_1(k,r) = \chi_1^-(k,r) + \chi_1^+(k,r)$$

$$\chi(k,r) \xrightarrow{r\to\infty} \chi_1$$

$$f(\theta,\varphi) = f$$

$$f_{DWBA}(\theta,\varphi) =$$



$$(E - H_0)\psi = V\psi \Longrightarrow \psi = (E - H_0)^{-1}\psi = G_0(E)V\psi$$

$$\psi = \phi + G_0 V\phi + G_0 V G_0 V\phi + \ldots$$

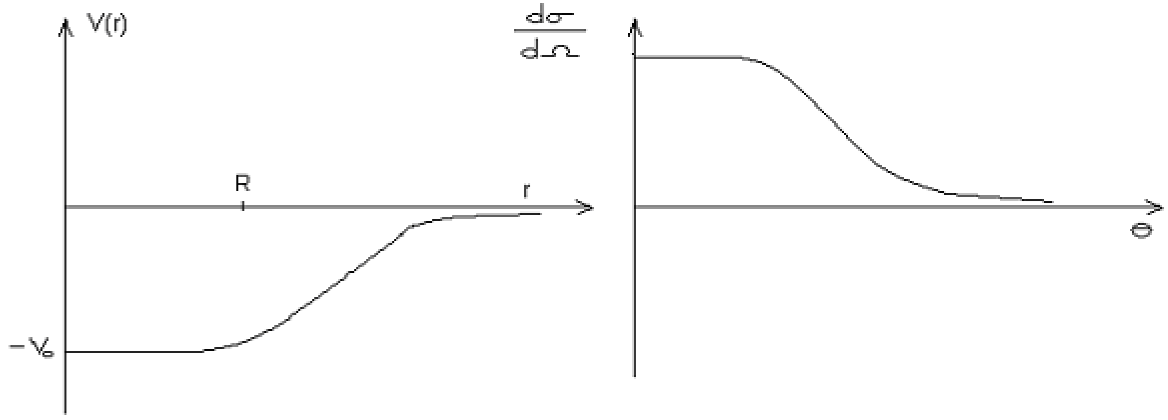
$$f(\theta,\varphi) = -\frac{m}{2\pi\hbar^2}\left[\int dr e^{-i\vec{k}'.\vec{r}} V(r) e^{i\vec{k}.\vec{r}} \int dr \int dr' e^{-i\vec{k}'.\vec{r}} V(r) G_0(r,r') V(r') e^{i\vec{k}.\vec{r}'} + \int dr \int dr' \int dr'' + \ldots\right]$$

Example: Gaussian Potential

$$V(r) = -V_0 e^{-(\frac{r}{R})^2} \qquad f(\theta) = \int_0^\infty V(r)\frac{\sin qr}{qr}4\pi r^2 dr \qquad q = 2k\sin(\frac{\theta}{2})$$

$$f(\theta) = -V_0\int_0^\infty e^{-(\frac{r}{R})^2}\frac{\sin qr}{qr}4\pi r^2 dr = -(2\pi)^{\frac{3}{2}}V_0 R^3 e^{-\frac{(qR)^2}{2}}$$

$$\frac{d\sigma}{d\Omega} = Ce^{-(2kR)^2\sin^2(\frac{\theta}{2})}$$



Schrödinger equation II: Partial wave methods

We must solve:

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}[E_{cm} - U_{\ell j}(r)]\right) u_{\ell j}(r) = 0$$

For bound states $E_{cm} < 0 \quad \kappa_b = \sqrt{\frac{2\mu|E_{cm}|}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) - \kappa_b^2\right) u_{n\ell j}(r) = 0$$

⇒ **Discrete Spectrum**

For scattering states $E_{cm} > 0 \quad k = \sqrt{\frac{2\mu E_{cm}}{\hbar^2}}$

$$\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\ell j}(r) + k^2\right) u_{k\ell j}(r) = 0$$

⇒ **Continuous Spectrum**

General Solution:

$$U_l(r) = F_l(r) + iG_l(r) + S_l[G_l(r) - iG_l(r)]$$

$F_l(r) = krj_l(kr)$ Bessel functions

$G_l(r) = -kr\eta_l(kr)$ Neumann functions

$f(\theta) = f_C(\theta) + \frac{1}{2ik}\sum_{l=0}^{l=\infty}(2l+1)(S_l - 1)e^{2i\sigma_l}P_l(\cos\theta)$ scattering amplitude

$$\sigma_{el} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)|1 - S_\ell|^2$$

$$\sigma_R = \frac{\pi}{k^2}\sum_l (2l+1)[1 - |S_l|^2]$$

⇒ $\sigma_{tot} = \sigma_{el} + \sigma_R$

Bessel and Neumann functions

For small x

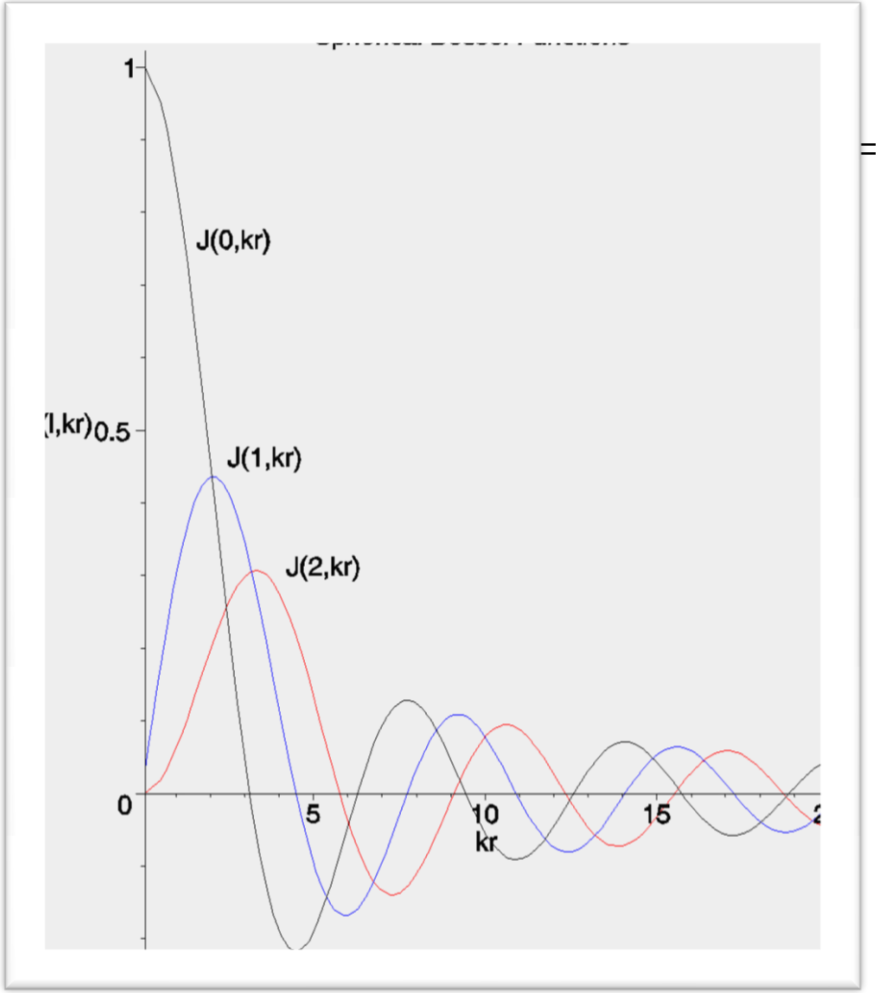
$$j_l(x) \to \frac{x^l}{(2l+1)!!}$$

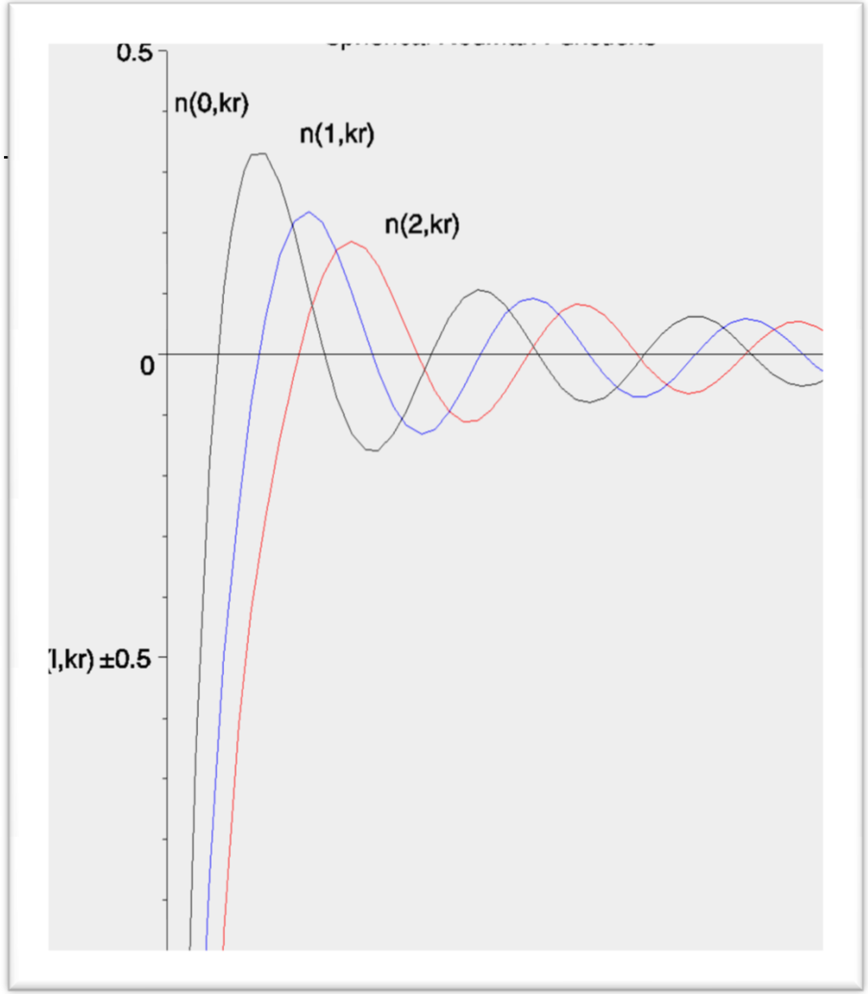
$$n_l(x) \to -\frac{(2l-1)!!}{x^{l+1}}$$

For large x

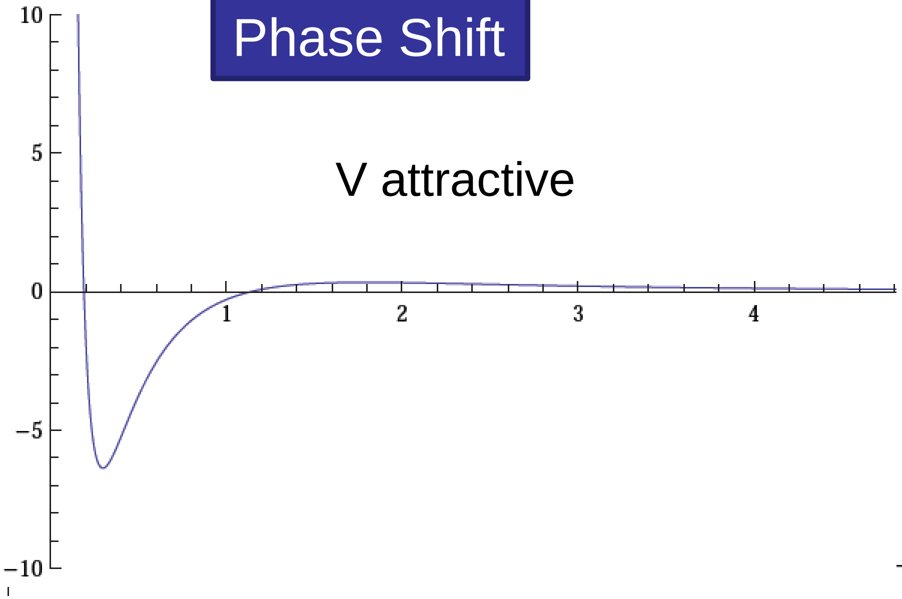
$$j_l(x) \to \frac{1}{x}\sin(x - l\pi/2)$$

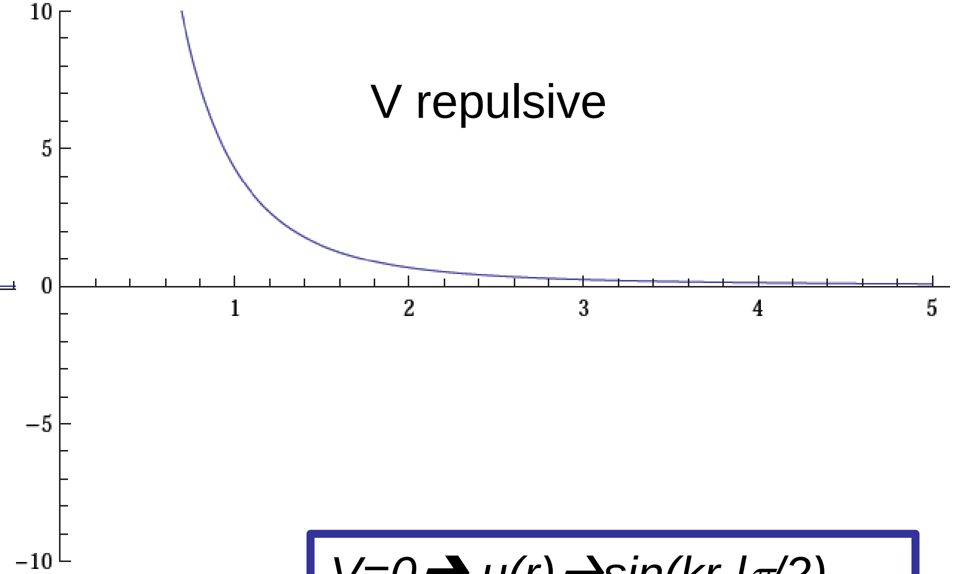
$$n_l(x) \to -\frac{1}{x}\cos(x - l\pi/2)$$



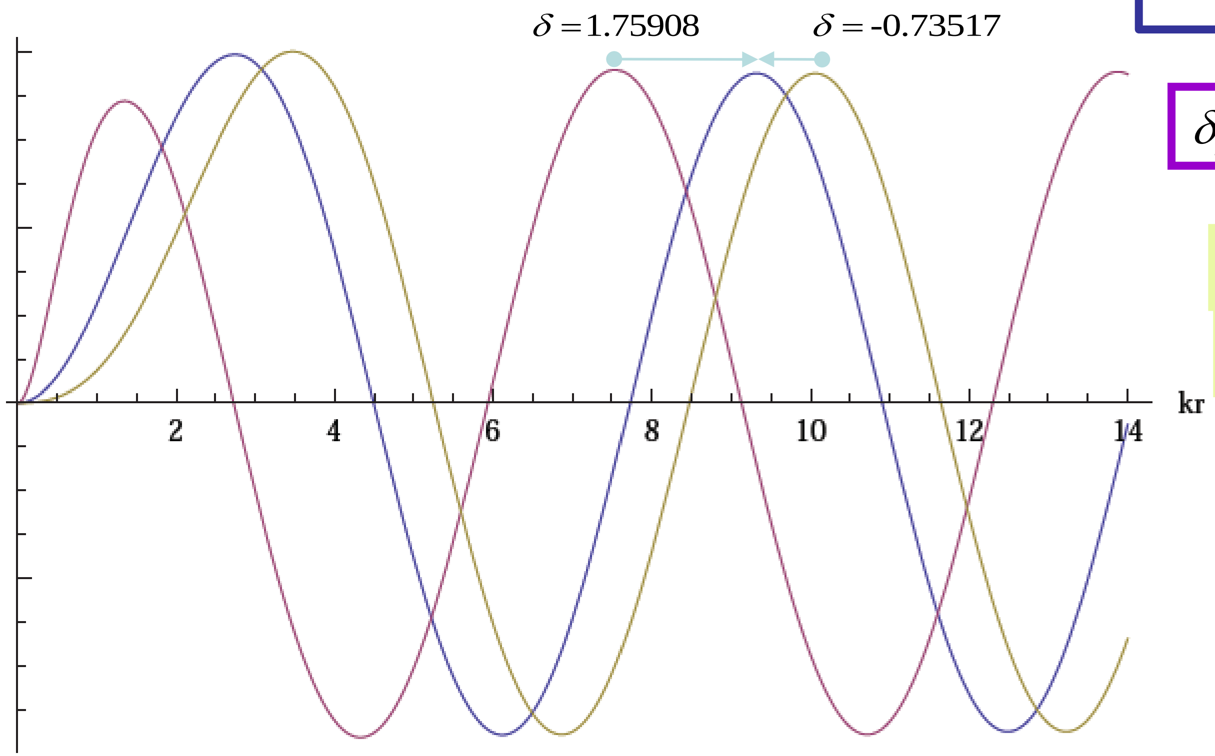


Phase Shift





$V=0 \rightarrow u(r) \rightarrow \sin(kr - l\pi/2)$
$\delta_l = 0$

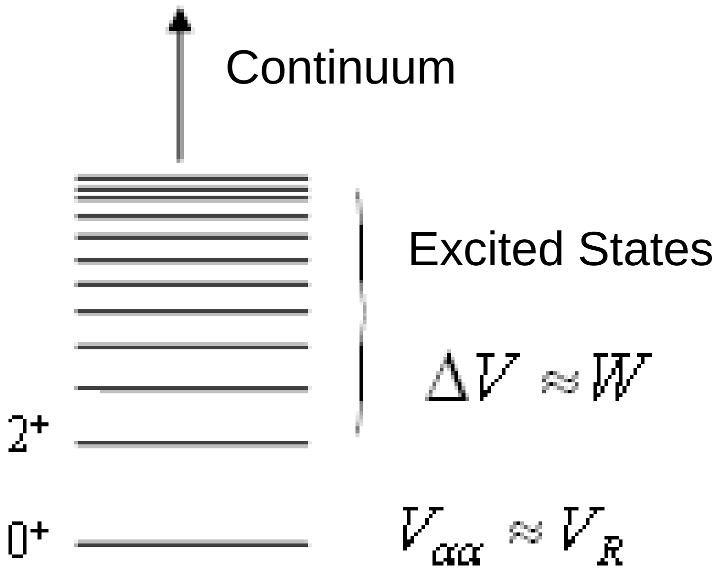


$\delta_l > 0 \rightarrow V$ *attractive*

$V \neq 0 \rightarrow u(r) \rightarrow \sin(kr - l\pi/2 + \delta_l)$

$\delta_l < 0 \rightarrow V$ *repulsive*

Optical Model



- Feshbach's formalism

$$V_{Nuclear} = V_{\alpha\alpha} + V_{\alpha\beta} \frac{1}{E^{(+)} - H_{\beta\beta}} V_{\beta\alpha} = V_{\alpha\alpha} + \Delta V(E) \approx V_R + iW$$

- Nuclear Potential:
  - Complex
  - Non-local
  - Energy and model space dependent
  - Resonant…

Model: General

- Optical Model (Elastic Scattering)

$$\left[\frac{\hbar^2}{2\mu}\frac{1}{r}\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}+k^2-\frac{L(L+1)}{r^2}\right)-(\alpha SLJ \mid V_\alpha \mid \alpha SLJ)\frac{1}{r}-\frac{V_C}{r}\right]\chi^J_{p,p}(k,r)=0$$

- Coupled-Channels Model (Elastic+Inelastic)

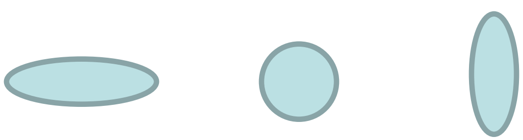
$$\left[\frac{\hbar^2}{2\mu}\frac{1}{r}\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}+k'^2-\frac{L'(L'+1)}{r^2}\right)-(\alpha' S' L' J \mid V_\alpha \mid \alpha' S' L' J)\frac{1}{r}-\frac{V_C}{r}\right]$$

$$\chi^J_{p',p}(k',r)=\sum_{p'\neq p''}(\alpha' S' L' J \mid V_\alpha \mid \alpha'' S'' L'' J)\frac{1}{r}\chi^J_{p'',p}(k'',r)$$

where $p \equiv \alpha L S$, $p' \equiv \alpha' L' S'$ $(\alpha SLJ \mid V_\alpha \mid \alpha SLJ) = V_{\alpha\alpha}$, $(\alpha' S' L' J \mid V_\alpha \mid \alpha'' S'' L'' J) = V_{\alpha\beta}$

- Deformation (Rotationel)

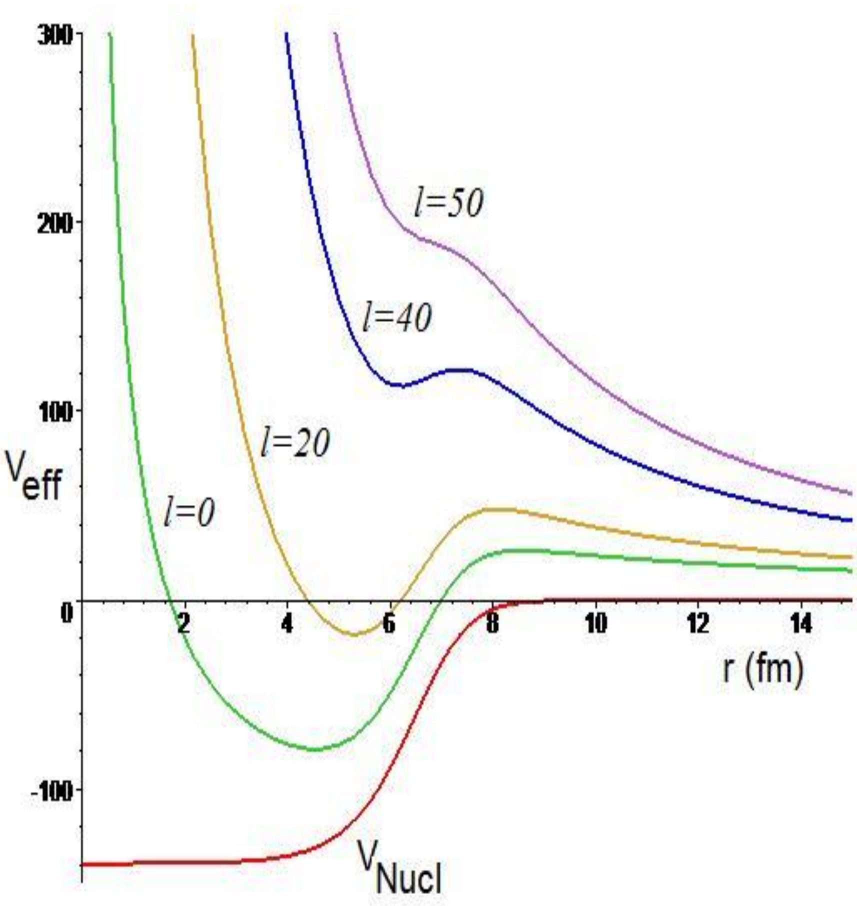
$$R = R_0[1+\beta_1 Y_{20}(\theta_1,\phi_1)+\beta_2 Y_{20}(\theta_2,\phi_2)]$$



$\beta < 0$ $\beta = 0$ $\beta > 0$

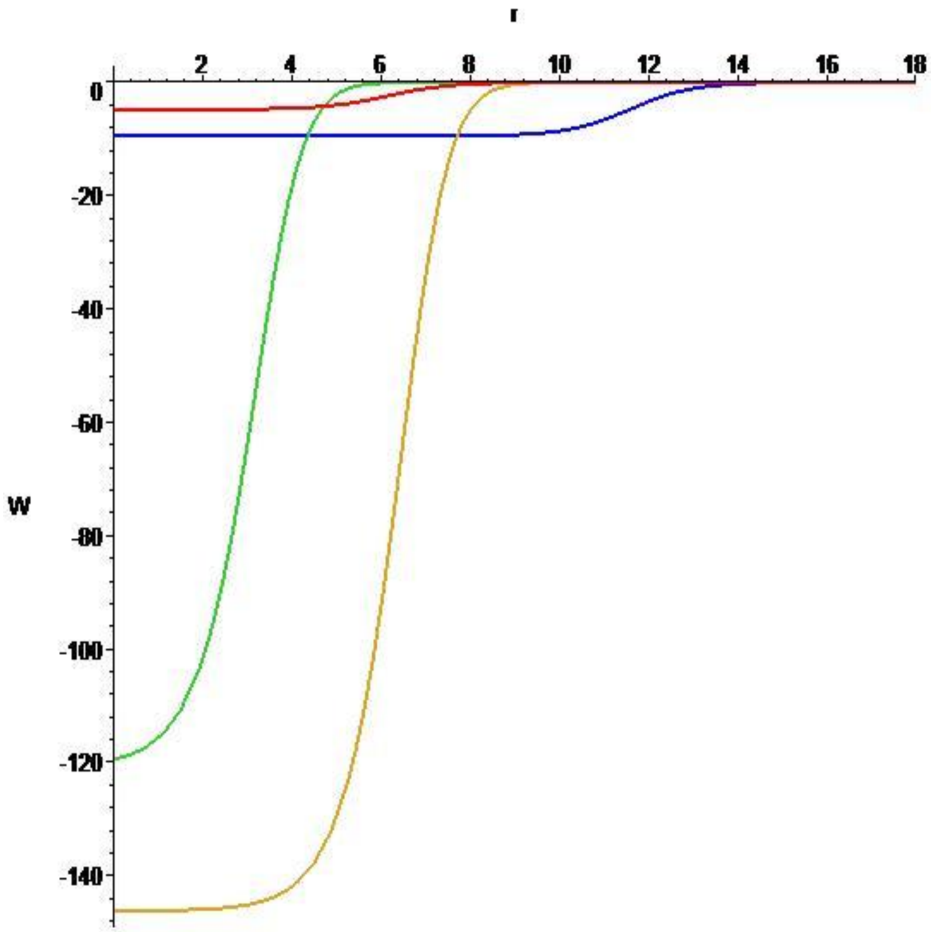
Optical Potentials

Real & Effective Real Potentials



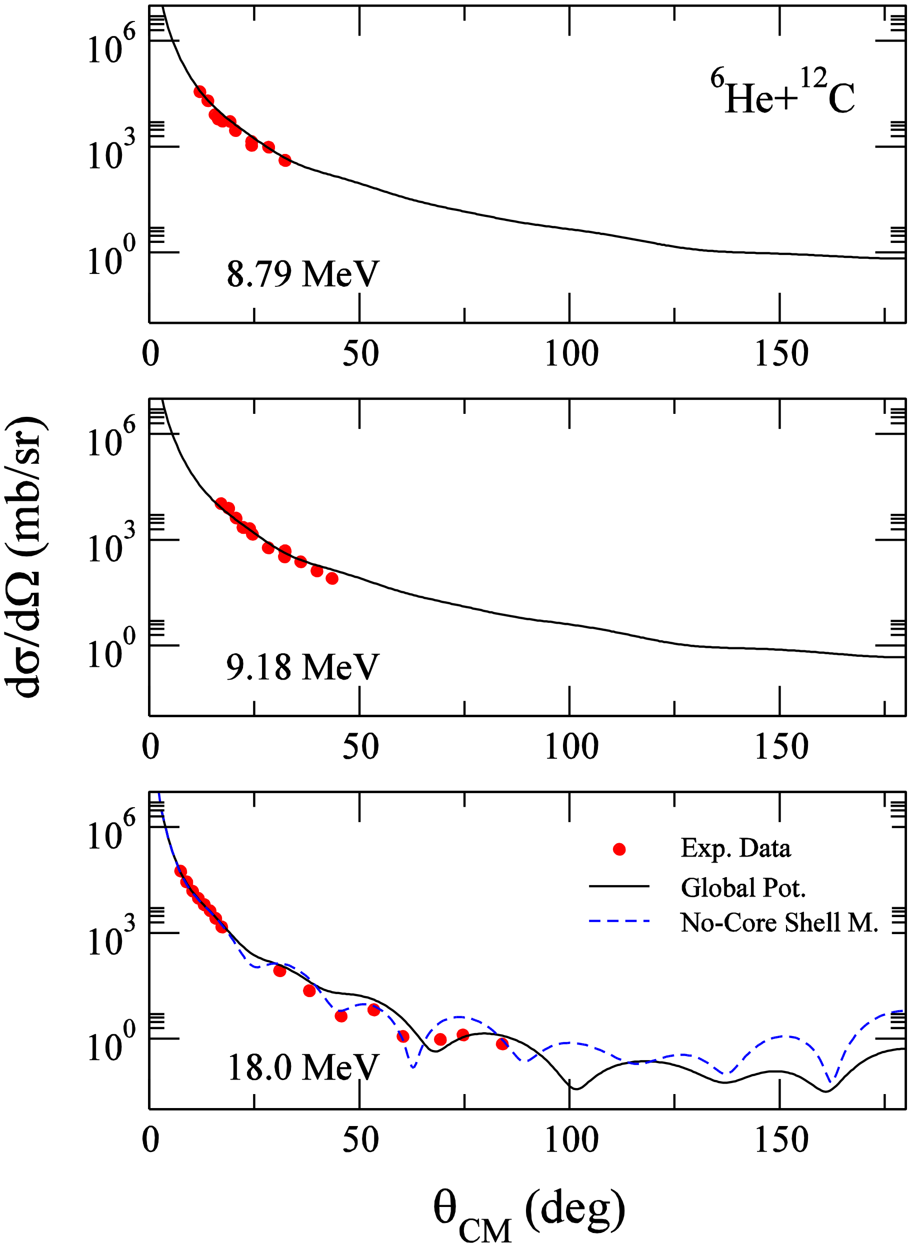
$^{6}He+^{208}Pb$ @ 18.0 MeV

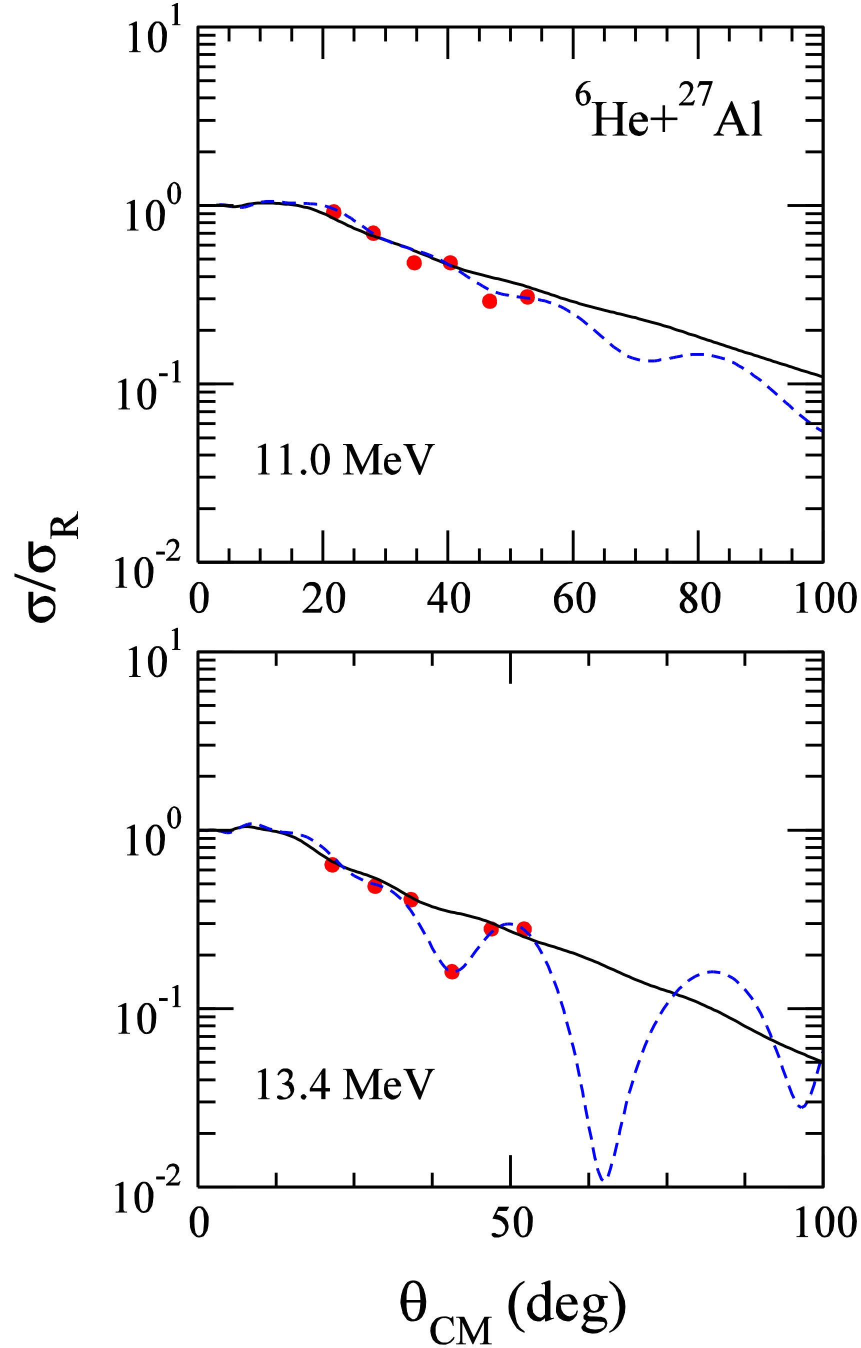
Imaginary & Real Potentials

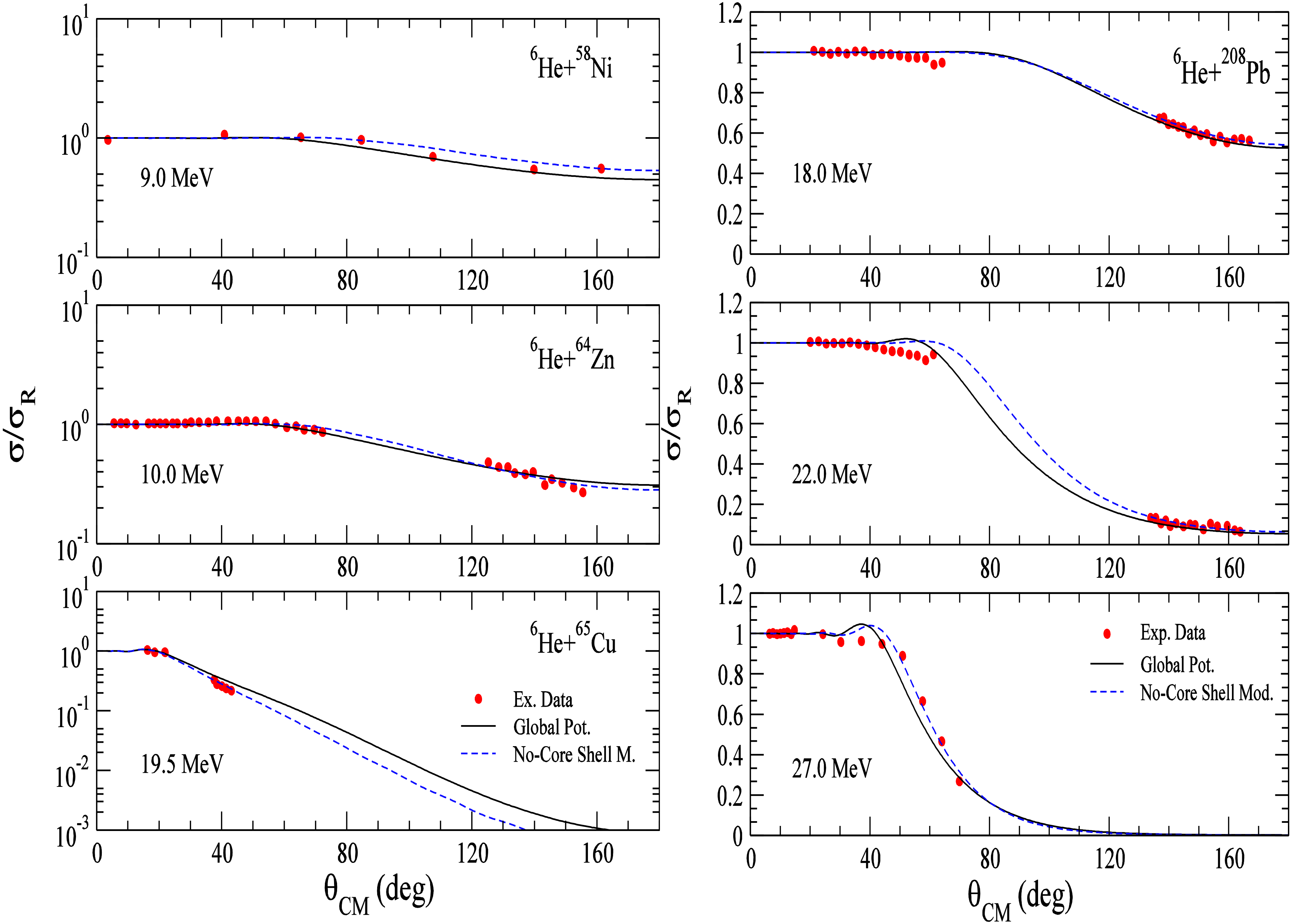


Long Range Absorption

Results

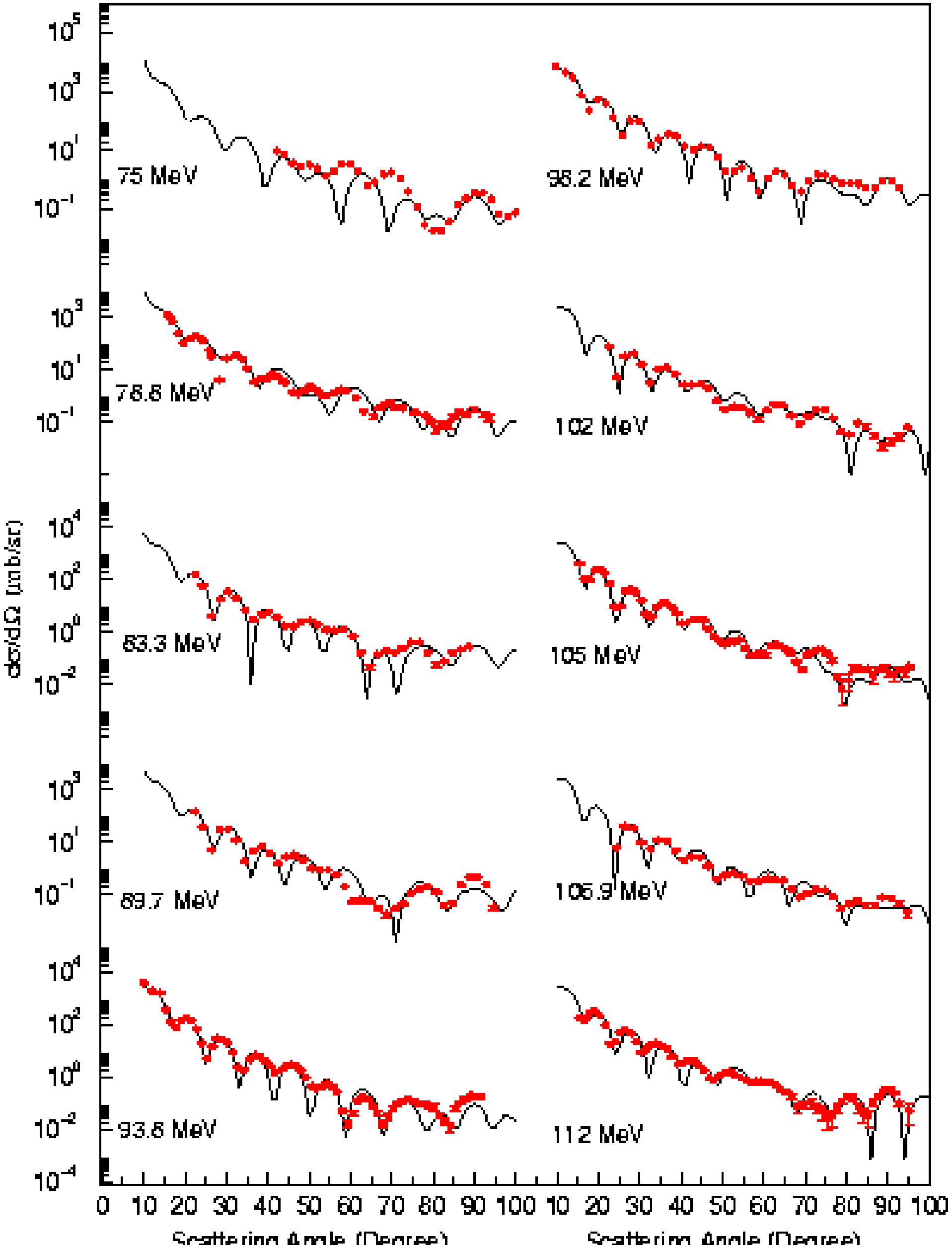

$^6He+^{12}C$
8.79 MeV
9.18 MeV
18.0 MeV
Exp. Data
Global Pot.
No-Core Shell M.
dσ/dΩ (mb/sr)
θ_{CM} (deg)


$^6He+^{27}Al$
11.0 MeV
13.4 MeV
σ/σ_R
θ_{CM} (deg)

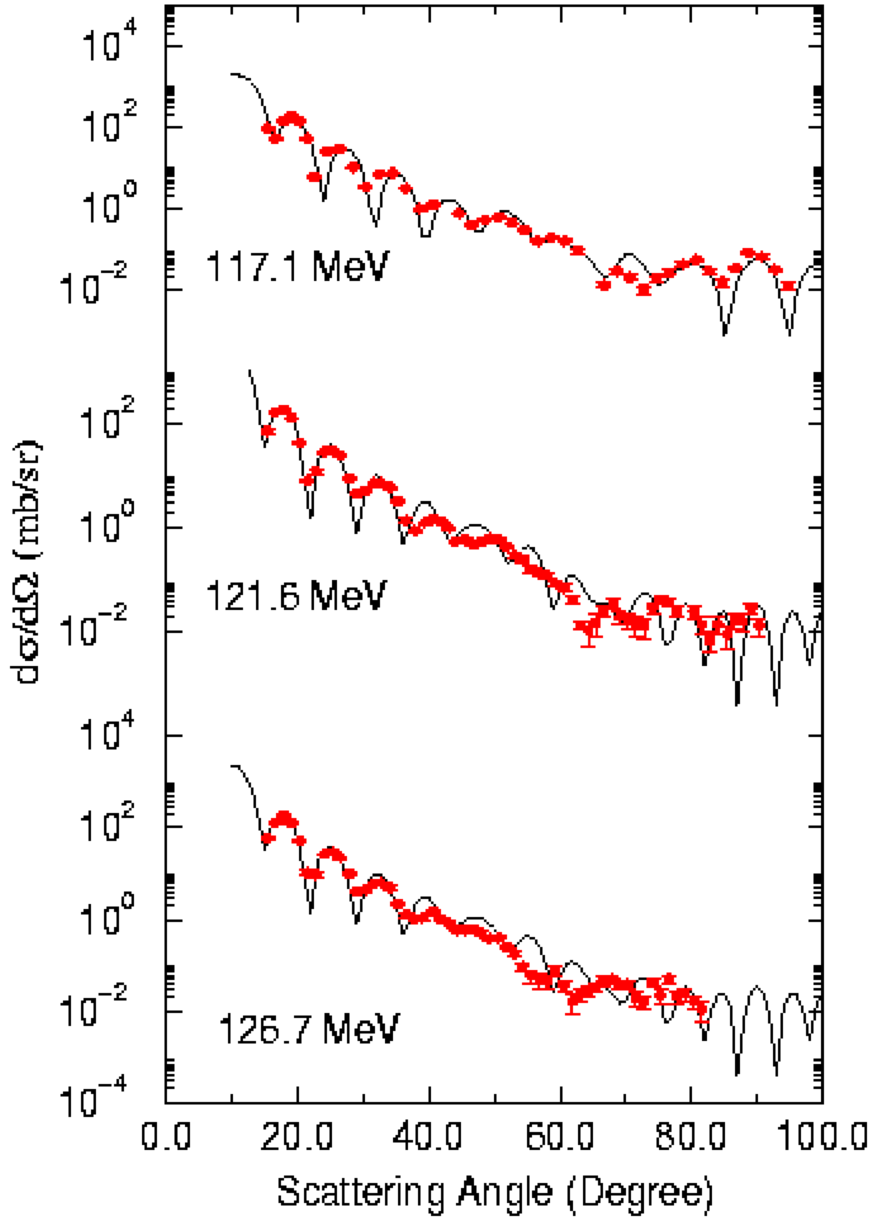


^{6}He+^{58}Ni
9.0 MeV
^{6}He+^{64}Zn
10.0 MeV
^{6}He+^{65}Cu
19.5 MeV
Ex. Data
Global Pot.
No-Core Shell M.
^{6}He+^{208}Pb
18.0 MeV
22.0 MeV
27.0 MeV
Exp. Data
Global Pot.
No-Core Shell Mod.
σ/σ_R
θ_{CM} (deg)

^{12}C-^{12}C

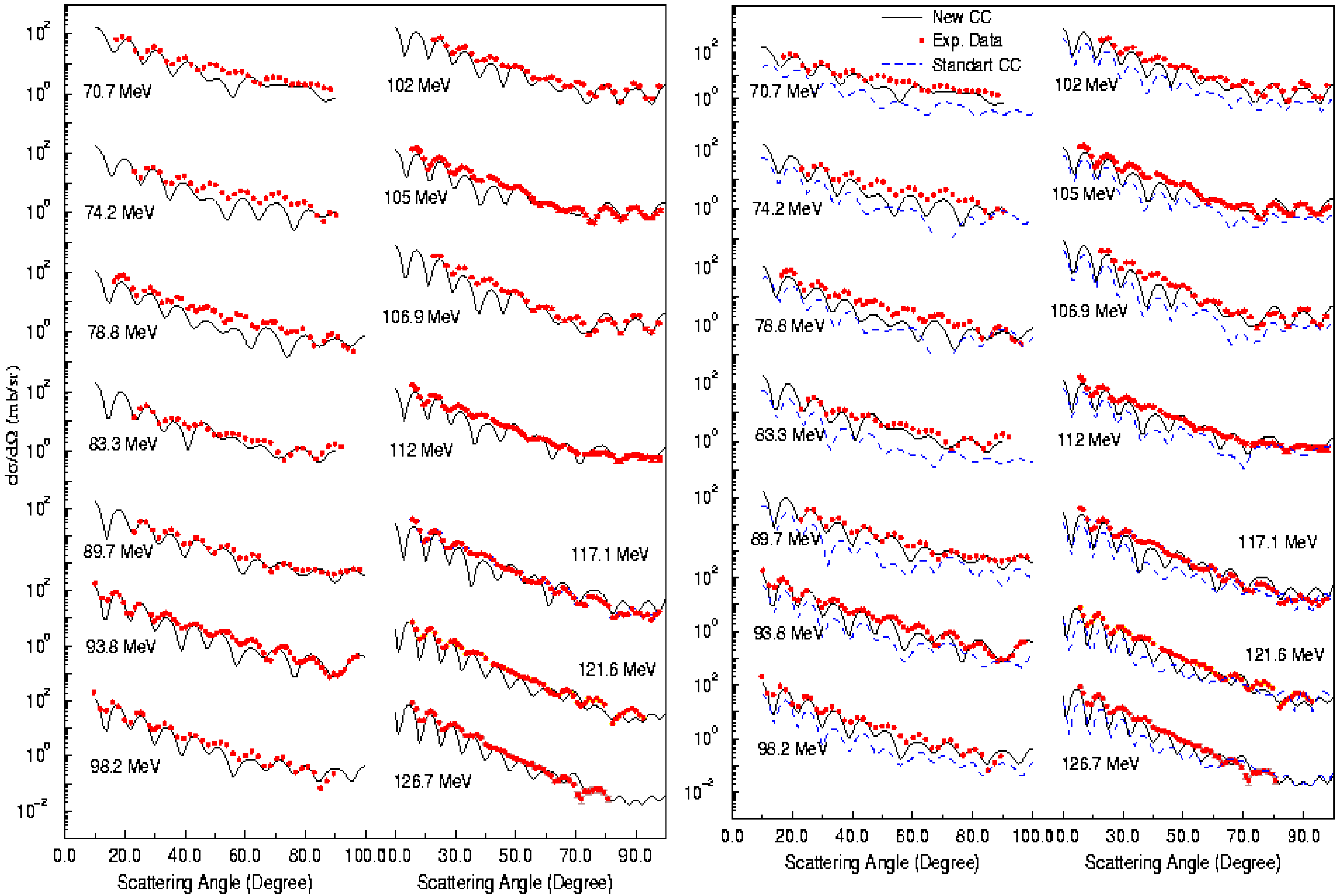


75 MeV
98.2 MeV
78.8 MeV
102 MeV
83.3 MeV
105 MeV
89.7 MeV
106.9 MeV
93.8 MeV
112 MeV
dσ/dΩ (mb/sr)
Scattering Angle (Degree)
Scattering Angle (Degree)



117.1 MeV
121.6 MeV
126.7 MeV
dσ/dΩ (mb/sr)
Scattering Angle (Degree)

^{12}C-^{12}C, 32-127.5 MeV Single-2^+



Thanks