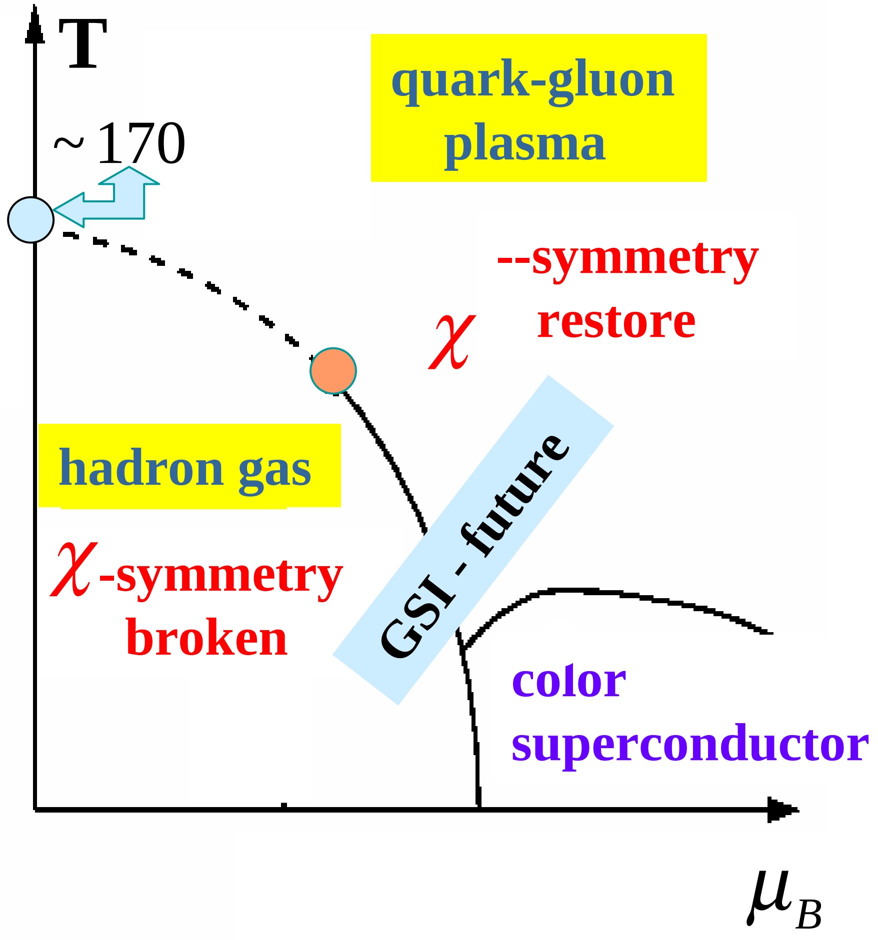
Hot and Dense QCD Matter

K. Redlich

- From LGT to heavy ion collisions
  - critical ⟺ freezeout conditions
  - equation of state and particle excitation functions
- Towards chiral symmetry resto-ration
  - in-medium hadronic spectral functions
  - dilepton production rate
- Color superconductor on the lattice in NJL-model



QCD at non-vanishing chemical potential $\mu_q > 0$

Bielefeld-Swansea approach

$$Z(V,T,\mu) = \int DA \det M(\mu) e^{-S(V,T)}$$

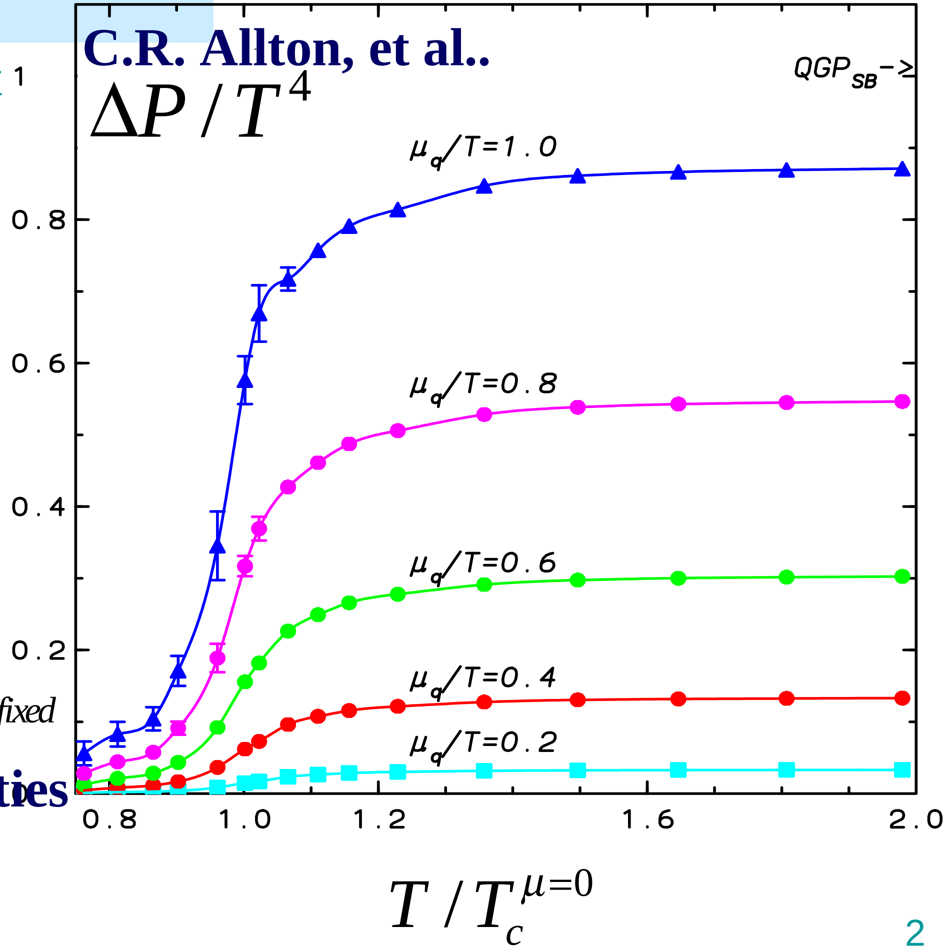
$$\Delta P = P(\mu) - P(0)$$

complex fermion determinant [1]

Taylor expansion of $P(\mu/T)$:

$$\frac{P(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T} \right)^{2n}$$

$$\frac{n_q}{T^3} = \left(\frac{\partial}{\partial(\mu/T)} \frac{P}{T^4} \right)_{T\ fixed}, \quad \frac{\chi_q}{T^2} = \left(\frac{\partial^2}{\partial(\mu/T)^2} \frac{P}{T^4} \right)_{T\ fixed}$$

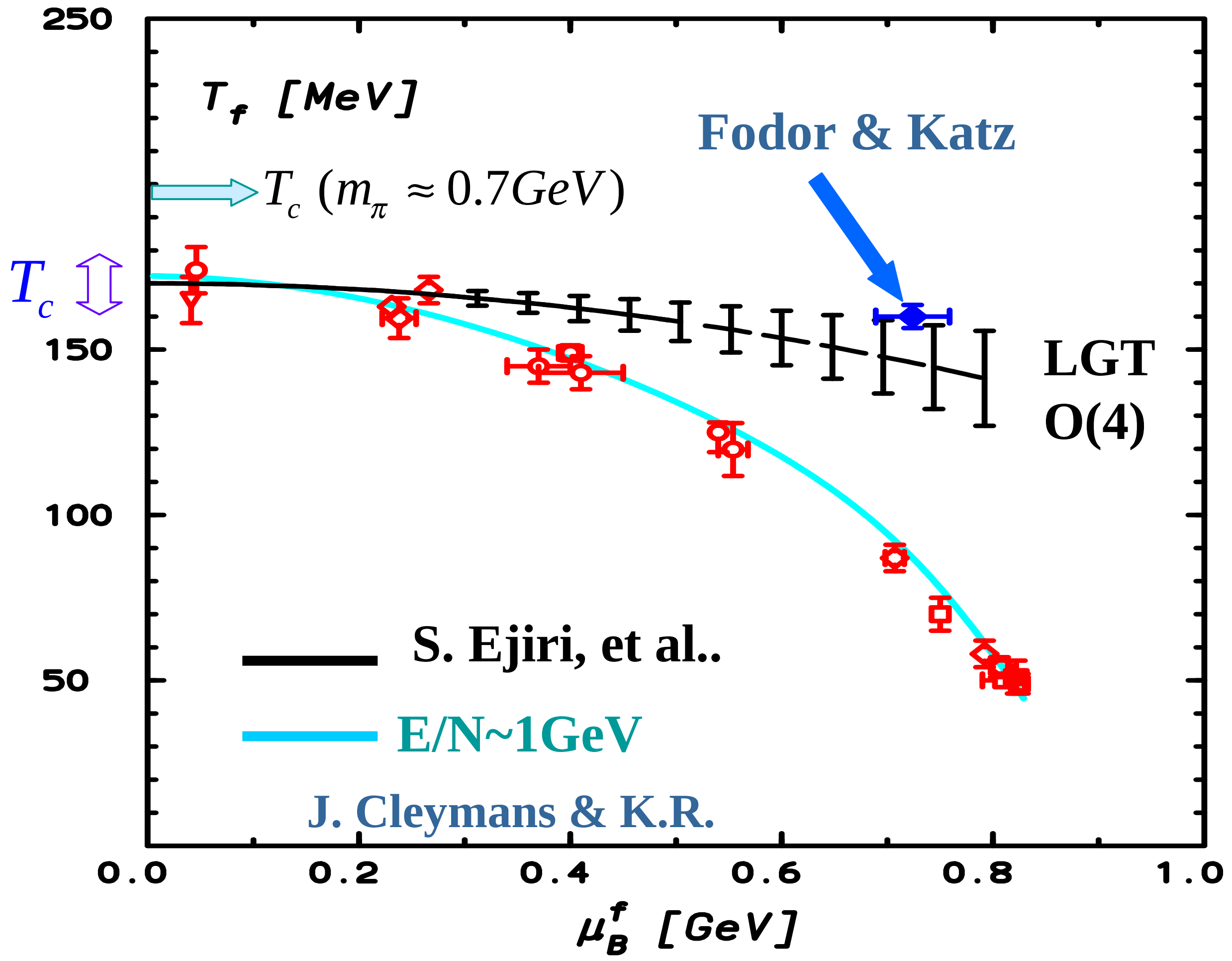


From μ dependence of chiral susceptibilities

$$\frac{T_c(\mu)}{T_c(0)} \approx 1 - \alpha(m_q) \left(\frac{\mu}{T_c(0)} \right)^2$$

Chemical freezeout curve from heavy ion data

$T_c^{\mu=0} \approx 173 \pm 8 [MeV]$ ⟺ **coincides with freezeout T at RHIC and SPC**



AGS, SPS, RHIC

J. Stachel, et al.

J. Cleymans, et al.

F. Becattini, et al.

P. Braun-Munzinger, et al.

N. Xu, et al..

SIS

H. Oeschler, et al..

R. Averbeck, R. Holzmann,

V. Metag, R. S. Simon

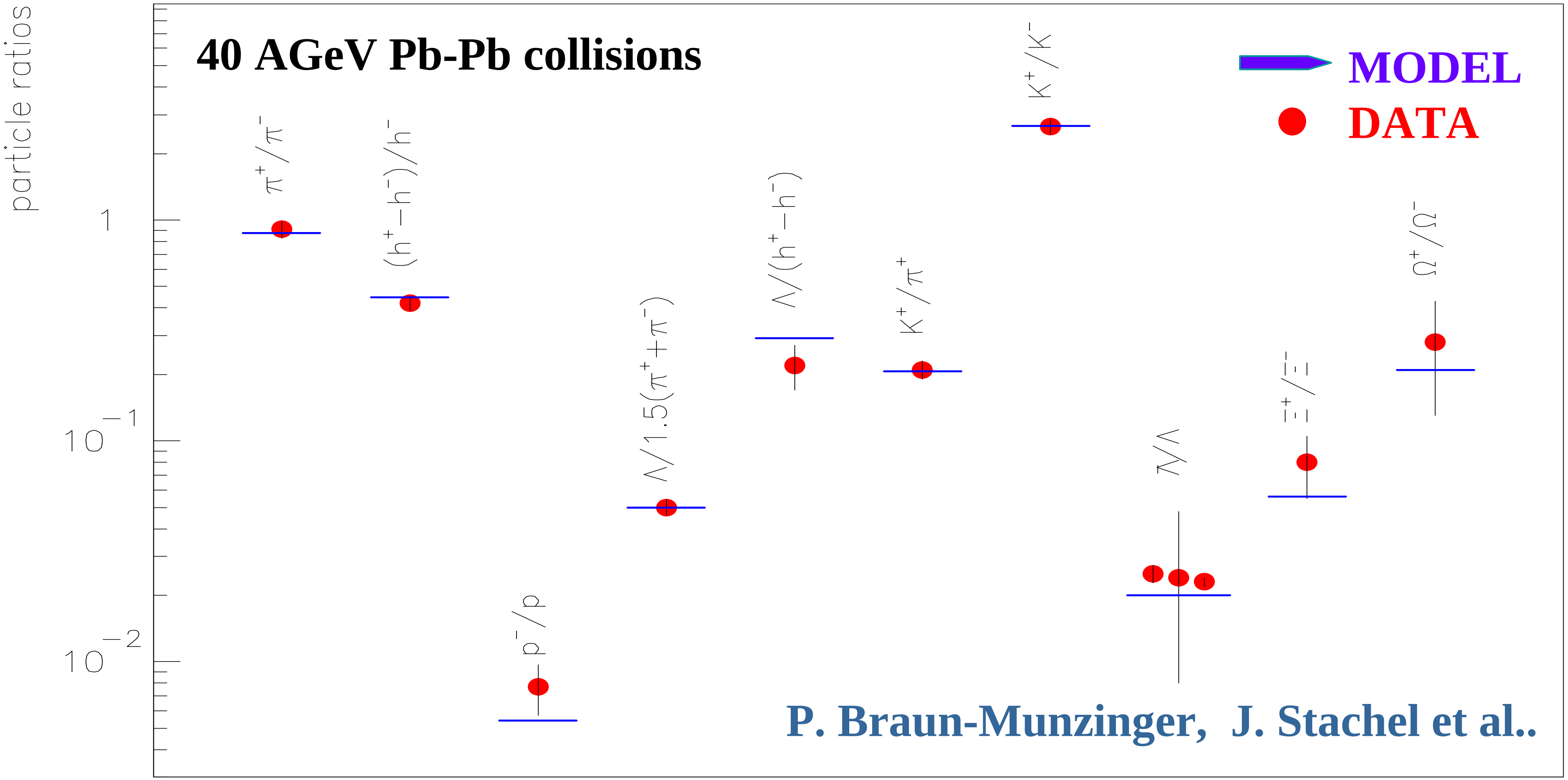
Thermal Freezeout

see recent results of

CERES Collaboration

$$Z_S = \frac{1}{2\pi}\int_{-\pi}^{+\pi} d\phi e^{-iS\phi} Tr[e^{-\beta(H-\mu_B B-\mu_Q Q-i\phi)}]$$

Only 2-parameters needed to fix all particle ratios



Taylor expansion of resonance pressure

Factorization of the baryonic pressure

$$\frac{P_B}{T^4} \approx F(T)\cosh\left(\frac{3\mu_q}{T}\right)$$

baryon mass spectrum

$$F(T) = \frac{1}{2\pi^2}\int dm\,\rho(m)\left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right)$$

Compare wich LGT results:

$$\frac{\Delta P}{T^4} \approx F(T)\left[c_2\left(\frac{\mu_q}{T}\right)^2 + c_4\left(\frac{\mu_q}{T}\right)^4\right]$$

$$\frac{n_q}{T^3} \approx F(T)\left[2c_2\left(\frac{\mu_q}{T}\right) + 4c_4\left(\frac{\mu_q}{T}\right)^3\right]$$

$$\frac{\chi_q}{T^2} \approx F(T)\left[2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2\right]$$

Consequences:

For fixed μ_q / T any ratio of these observables is T-independent

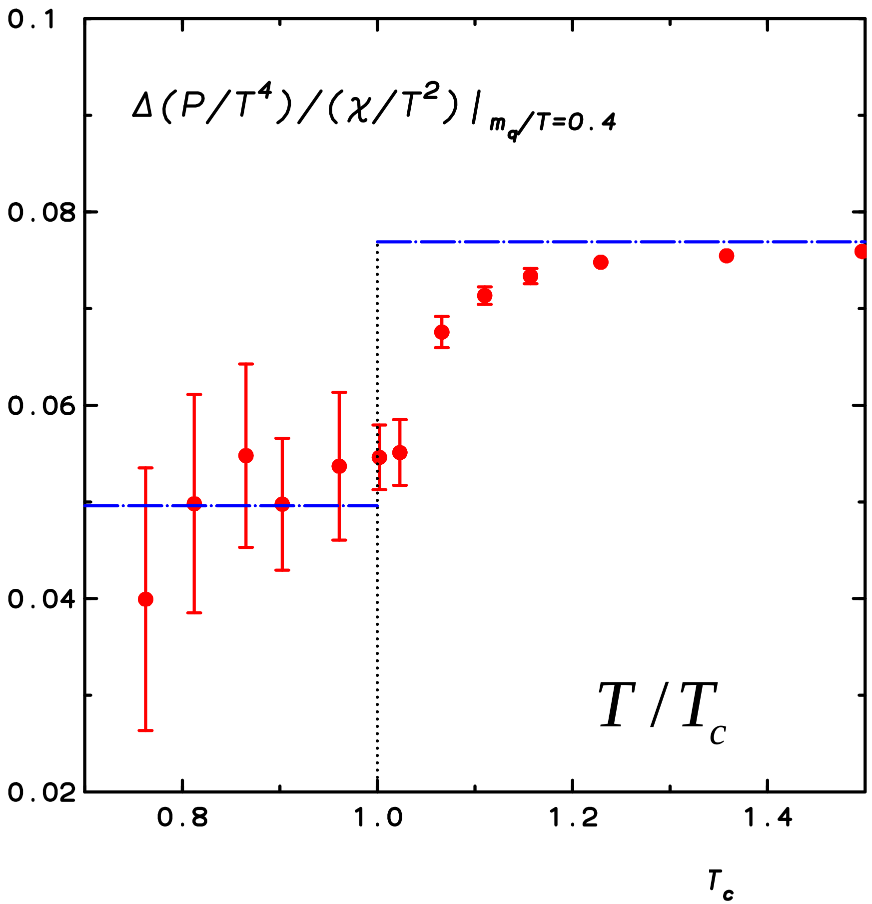
the ratio of the O(2) and O(4) coefficients: $\dfrac{c_4}{c_2} = \dfrac{3}{4}$

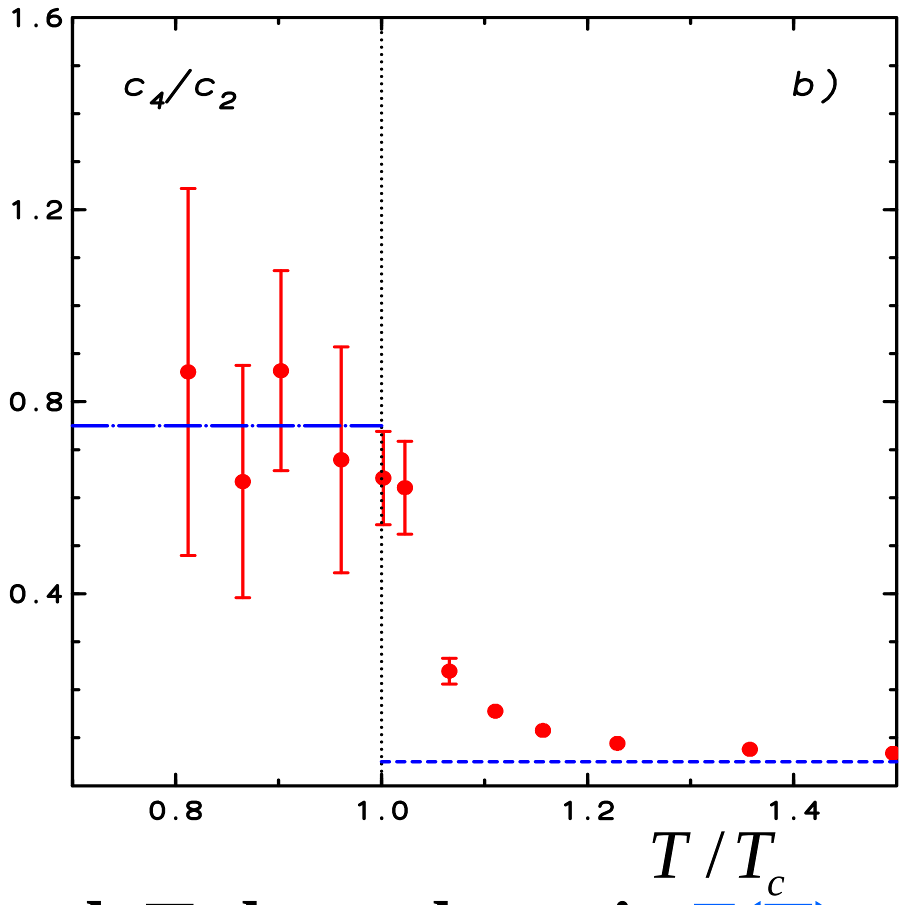
QCD partition fuction from LGT and Phenomenology

F. Karsch, A. Tawfik, K.R.

μ / T — factorization

Taylor cooficients of cosh(x)





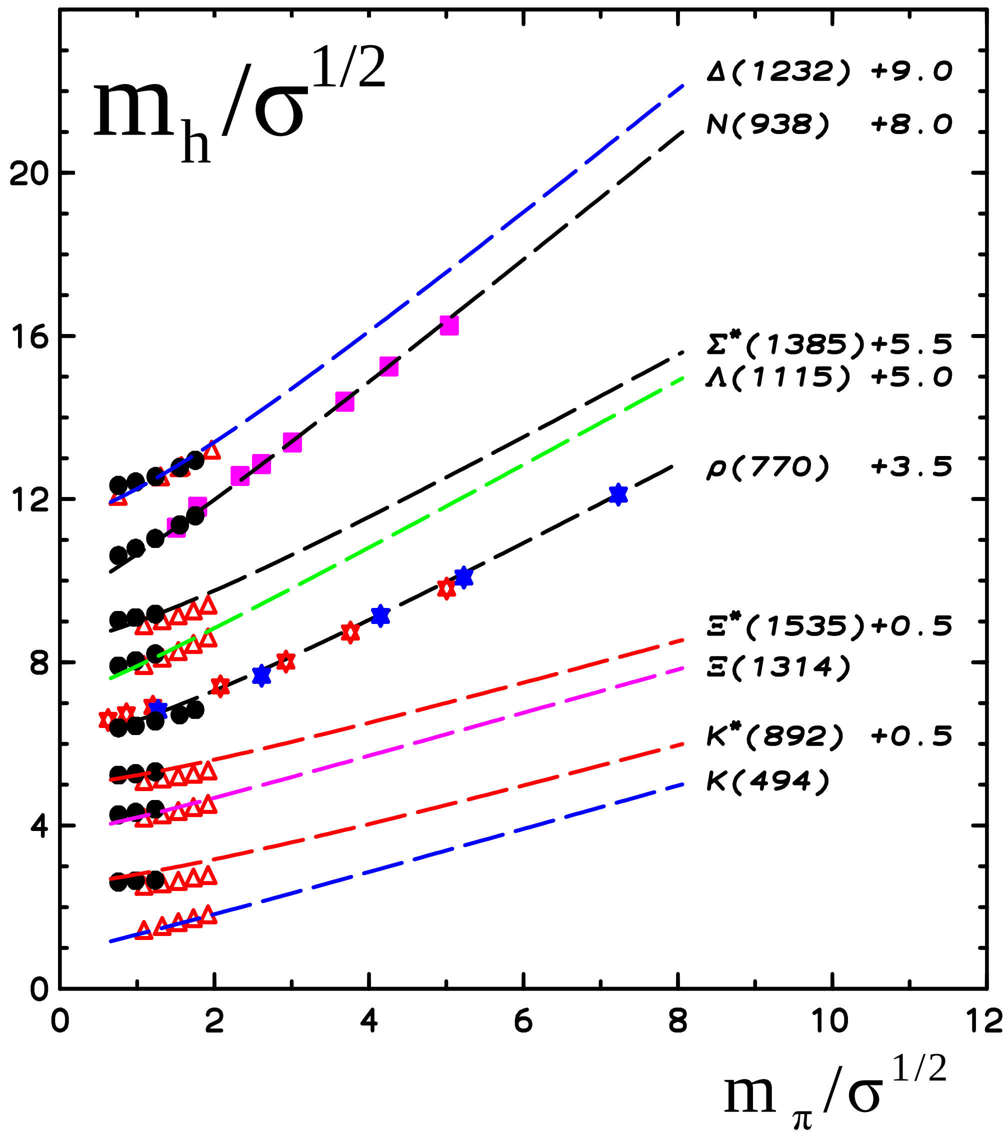
$$\frac{P_B}{T^4} \approx F(T)\cosh\left(\frac{3\mu_q}{T}\right)$$

⇒ check T-dependence in F(T)

required: $m_{hadron} = f(m_{quark})$

Hadron Mass Spectrum – LGT and Bag model results

F. Karsch, A. Tawfik, K.R.



LGT results for pion mass dependence of N, Δ and their parity partners

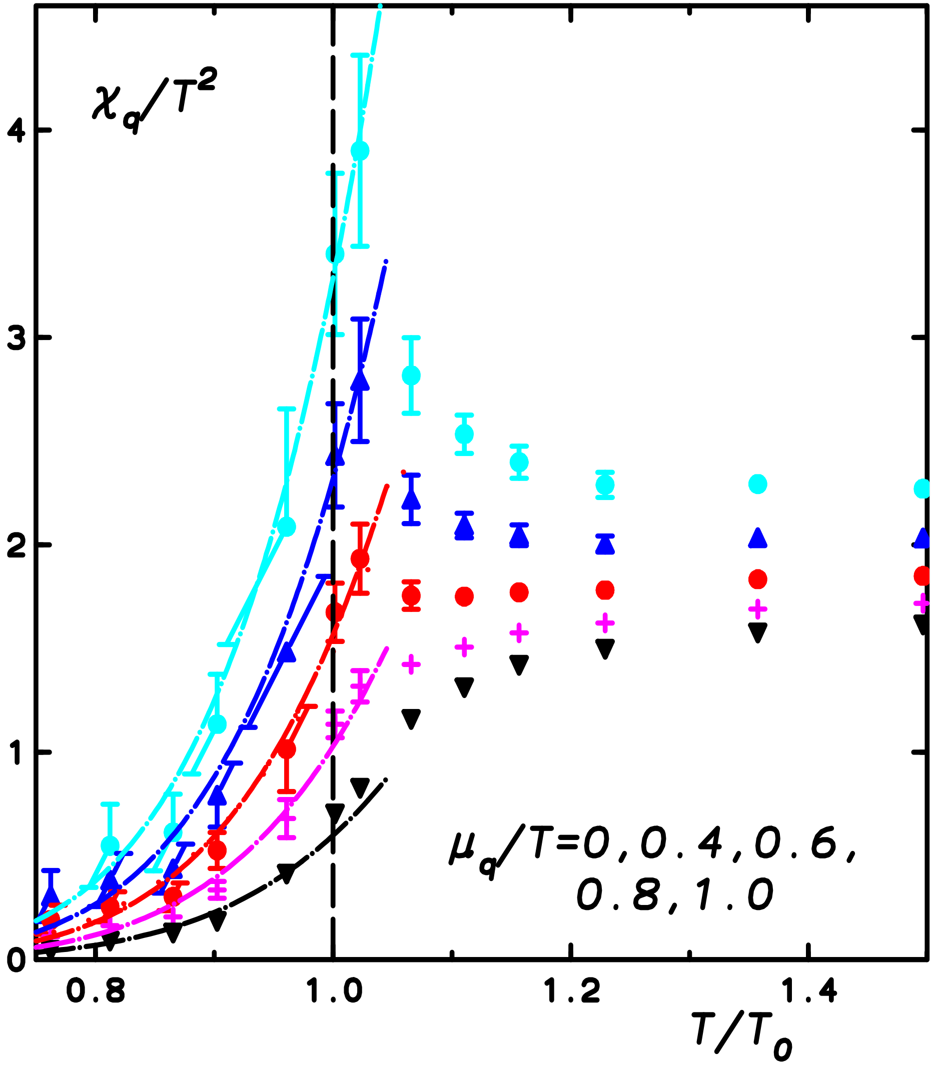
$$\frac{m_h(m_\pi)}{m_h^{phys.}} \approx 1 + A\left(\frac{m_\pi}{m_h^{phys.}}\right)^2$$

$$A = 1 \pm 0.1$$

QCDSF Coll., M. Göckeler, et al..

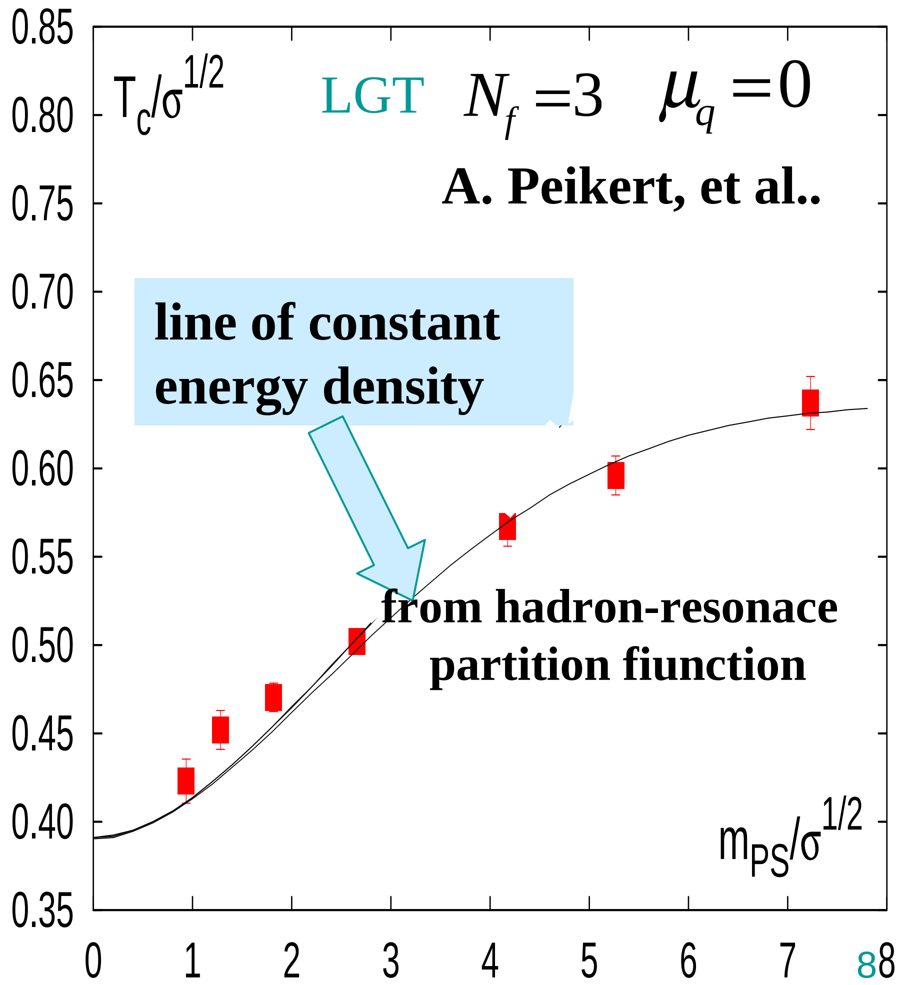
Deconfinement is density driven - (percolation)

$$\frac{\chi_q^{QGP}}{T^2} = N_f \left[1 + \frac{3}{\pi^2} \left(\frac{\mu_q}{T} \right)^2 \right]$$

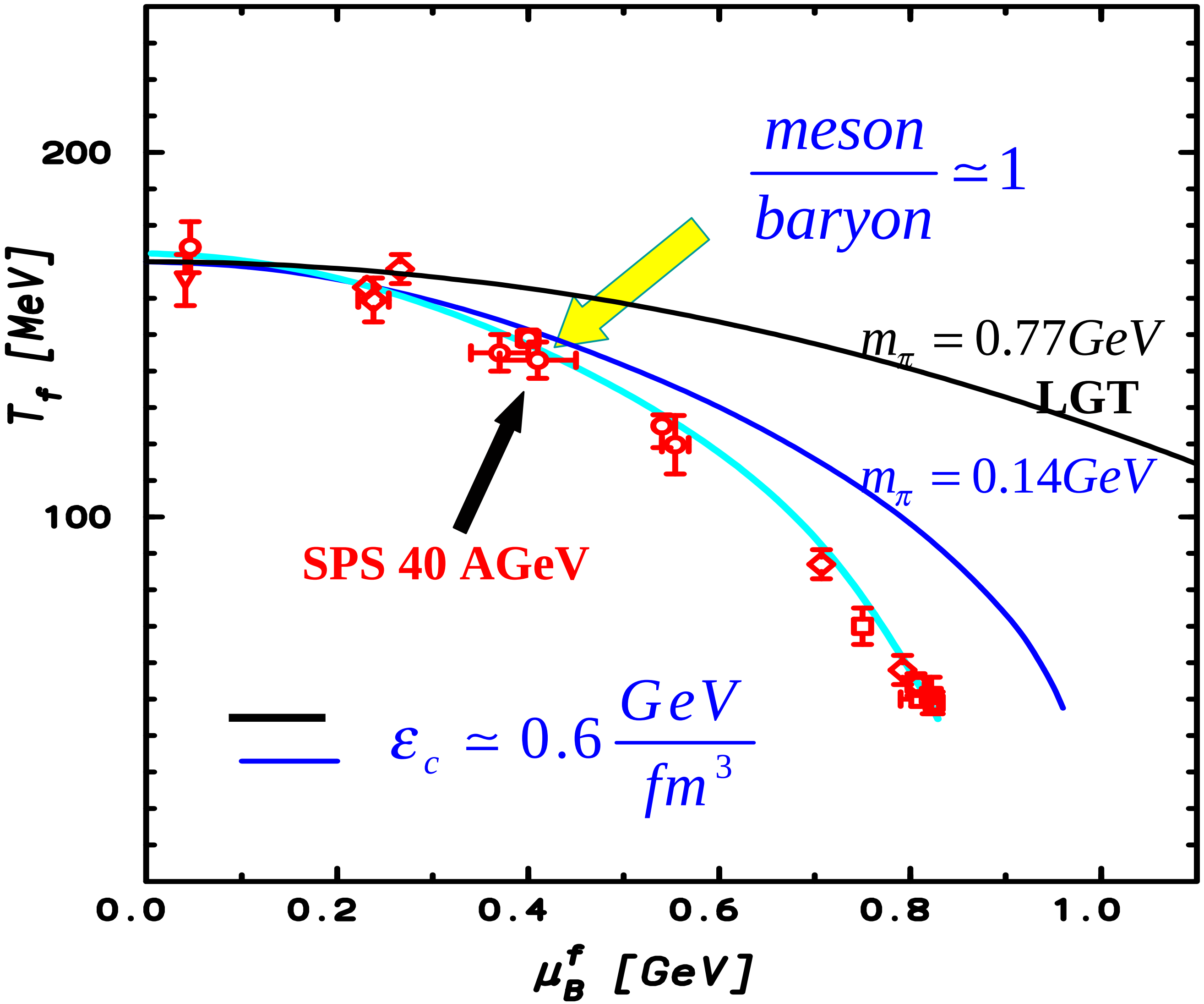


Hadron resonace gas partition function

good descritpion of QCD thermodynamics and critical conditions



Phase boundary of fixed energy density versus chemical freezeout



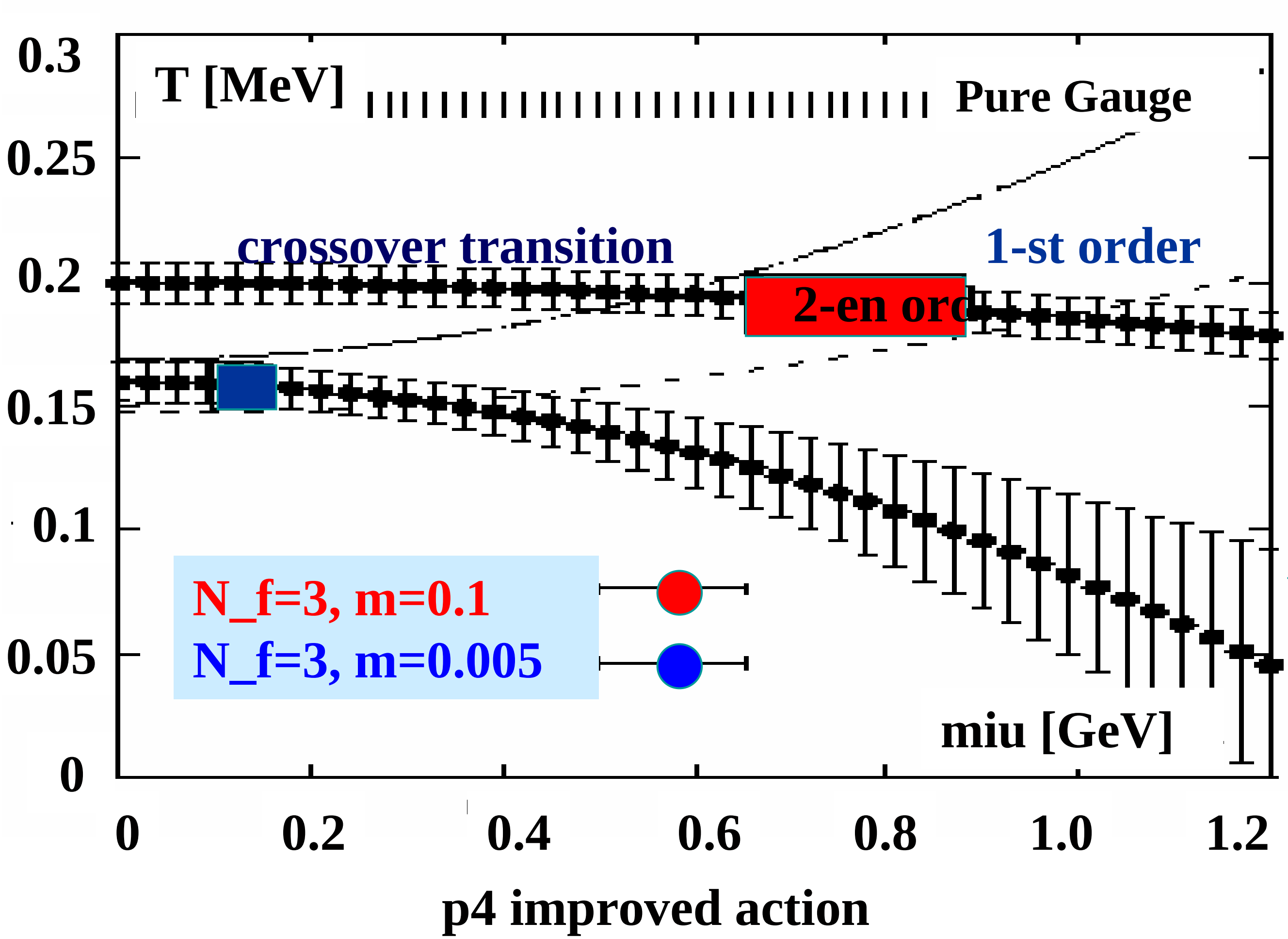
- Spliting of chemical freezeout and phase boundary surface appears when the **densities of mesons and baryons are comparable**
- For E<40 AGeV strong collective effects in hadronic medium are to be expected thus,
- extrapolation of critical condition of fixed ε calculated with free particle dispersion relation can be only a crude approximation

see also NJL results on critical conditions (T. Kunihiro et al.)

Chiral critical point in 3-flavour QCD

F. Karsch et al.

Strong dependence of the position of second order endpoint on the quark mass!



$$\frac{T_c(\mu)}{T_c(0)} \approx 1 - \alpha(m_q)\left(\frac{\mu}{T_c(0)}\right)^2$$

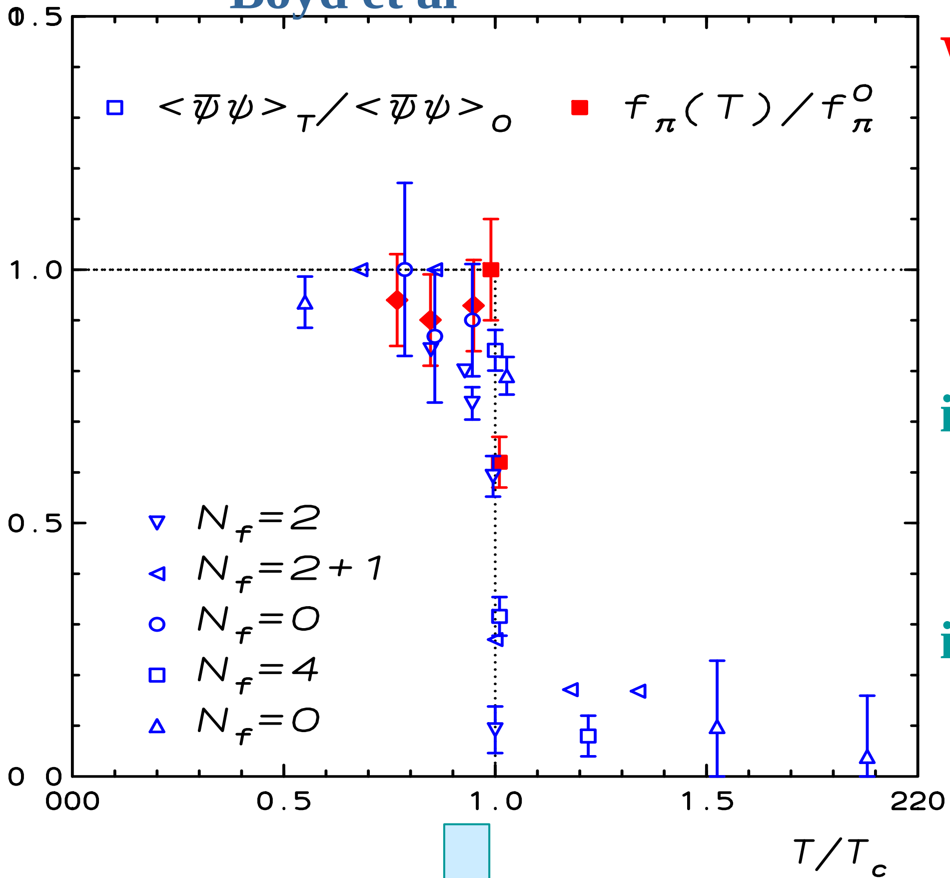
$$\alpha = \begin{pmatrix} 0.025 & m_q a = 0.1 \\ 0.114 & m_q a = 0.005 \end{pmatrix}$$

Strong dependence of the slope on the quark mass

Chiral Symmetry Restoration

need in-medium spectral function

Boyd et al



critical region very narrow

Weinberg sum rules: (Kapusta &Schuryak)

$$f_\pi = -\frac{1}{\pi}\int \frac{ds}{s}\left(\mathrm{Im}\,\Pi_\rho - \mathrm{Im}\,\Pi_{a_1}\right)$$

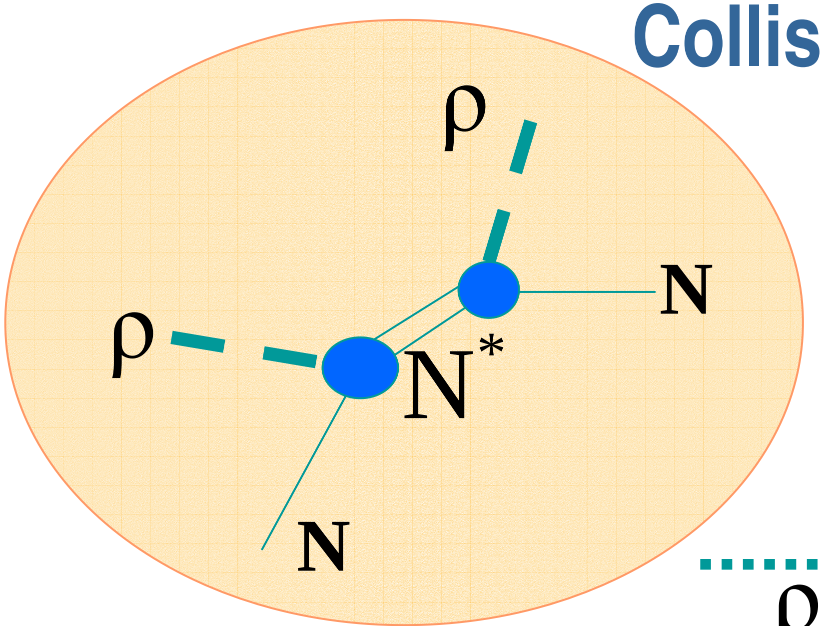
in vacuum: spontaneous chir. sym. breaking

$$f_\pi \neq 0 \iff \mathrm{Im}\,\Pi_\rho \neq \mathrm{Im}\,\Pi_{a_1}$$

in medium (phase tran.): chir. sym. restor

$$f_\pi = 0 \iff \mathrm{Im}\,\Pi_\rho = \mathrm{Im}\,\Pi_{a_1}$$

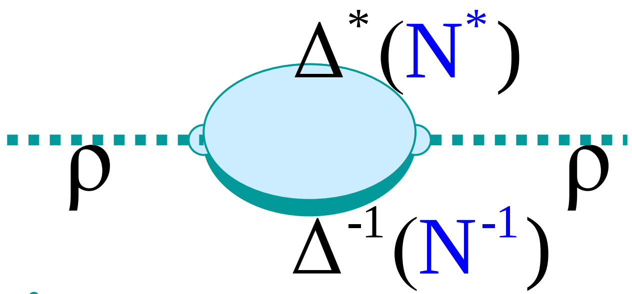
Collision broadening – vector mesons



$$D_h^{-1} = \omega^2 - \vec{p}^{\,2} - m_h^2 - \Sigma_h\left(\omega, \vec{p}\right)$$

pion cloud modification

direct interactions with hadrons

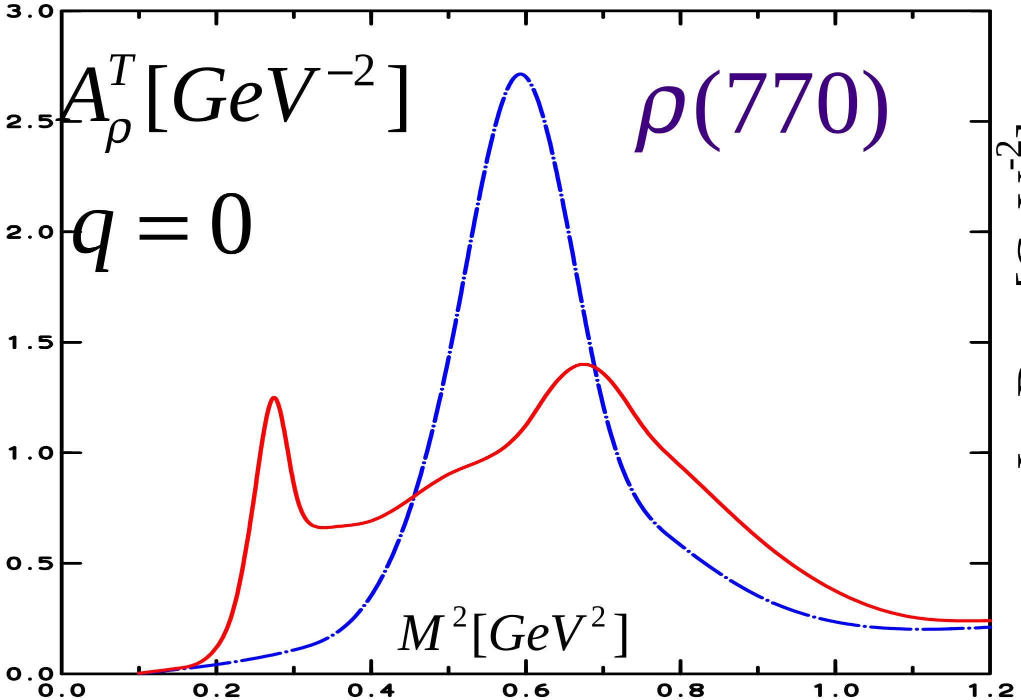


B. Friman
& H. Pierner

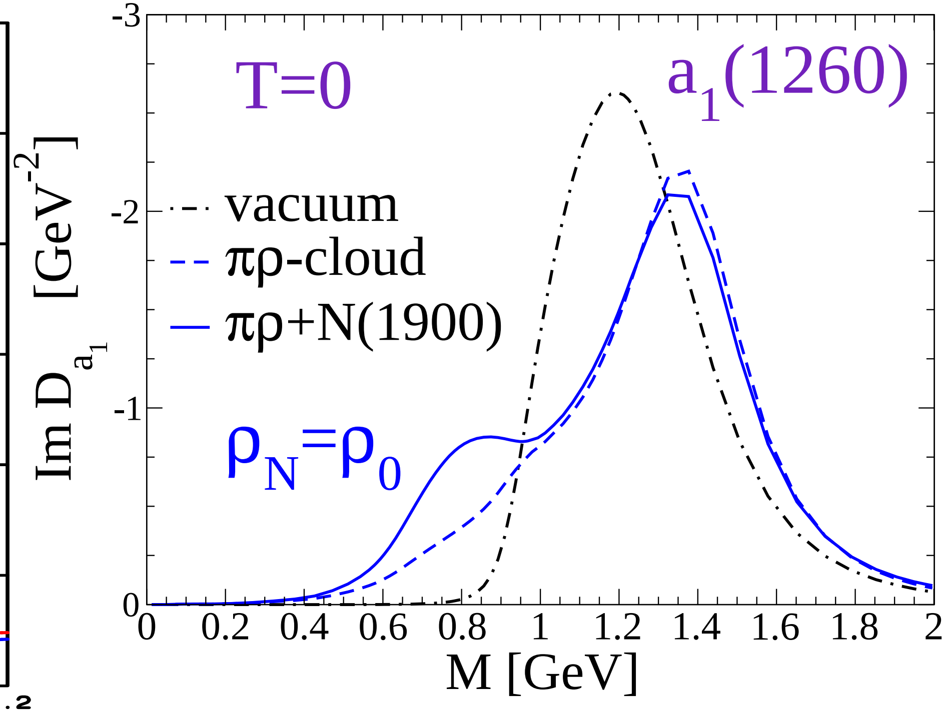
M. Post, S. Leupold, U. Mosel;
M. F.M. Lutz, G. Wolf,
B. Friman;
M. Urban, M. Buballa, J. Wambach
R. Rapp, et al..; W. Weise, et al..;
T. Hatsuda, S.H. Lee;
Ch. Gale, et al..; G. Chanfray et al..; G.E. Brown, et al..; ...

Vector meson spectral function
effective Lagrangian approach

M. Post, S. Leupold, U. Mosel



R. Rapp et al..



low M => significant contribution from couplings of:

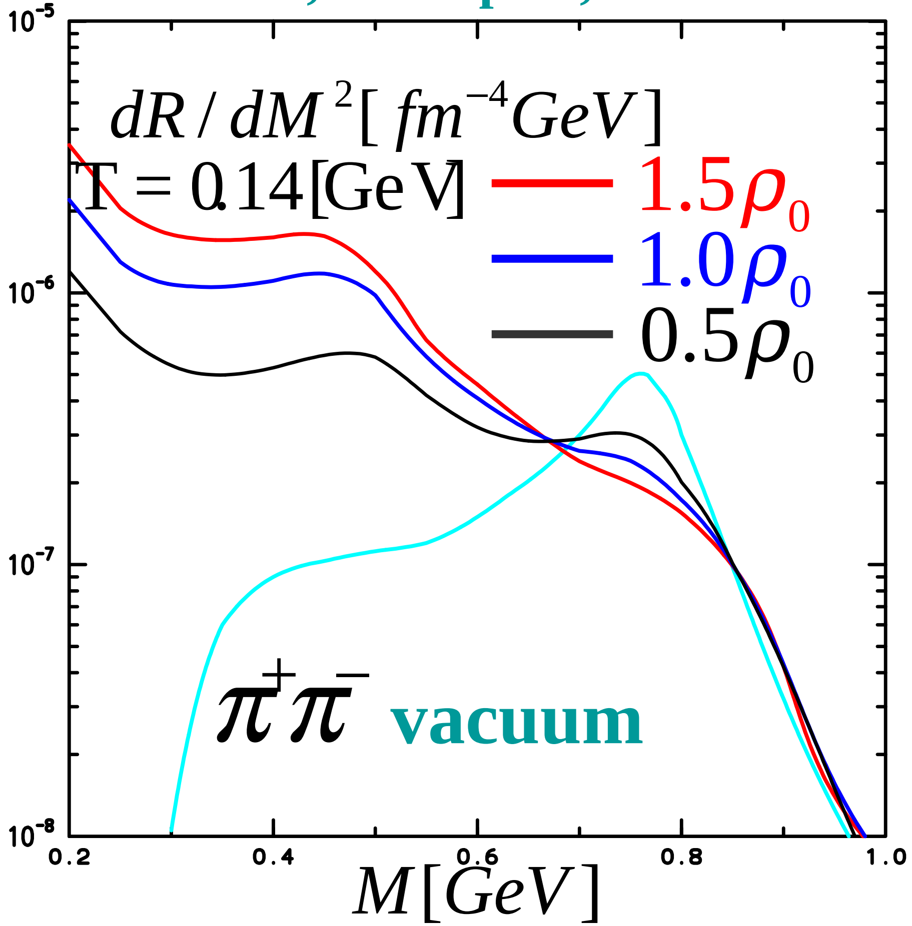
ρ **to** the $N^*(1520)N^{-1}$ and a_1 **to the** $N^*(1900)N^{-1}$ **states**

In medium effects and dilepton yield

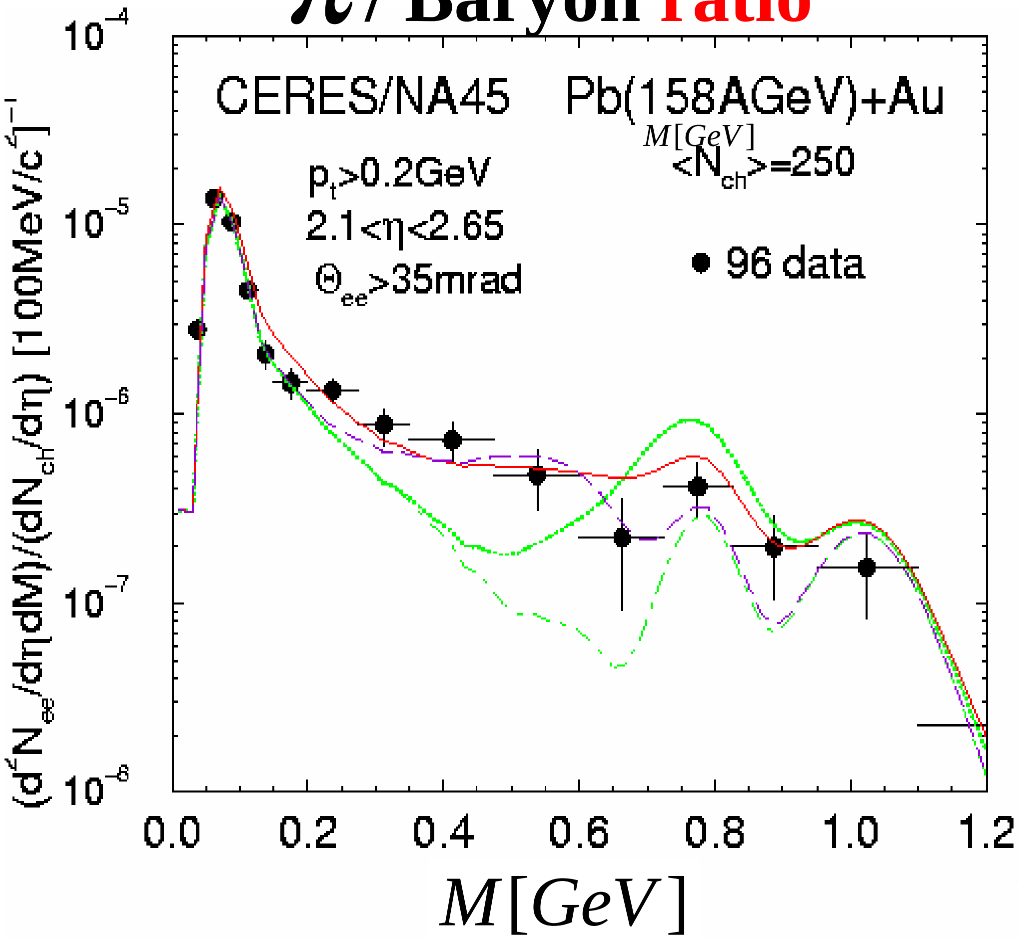
$$\frac{dR_{ee}}{d^4q} = -\frac{\alpha^2}{\pi^3 M^2} f(q_0, T)\,\mathrm{Im}\,\Pi^{em}(M, q, T, \mu)$$

1-loop $(m_\rho^4 / g_\rho^2)\,\mathrm{Im}\,G_\rho$

M. Post, S. Leupold, U. Mosel



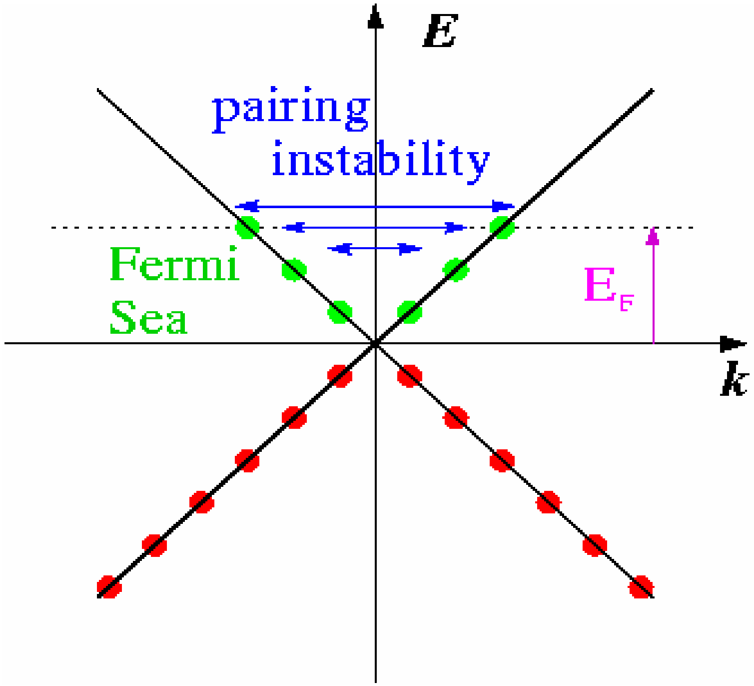
enhancement increases with decreasing π / Baryon ratio

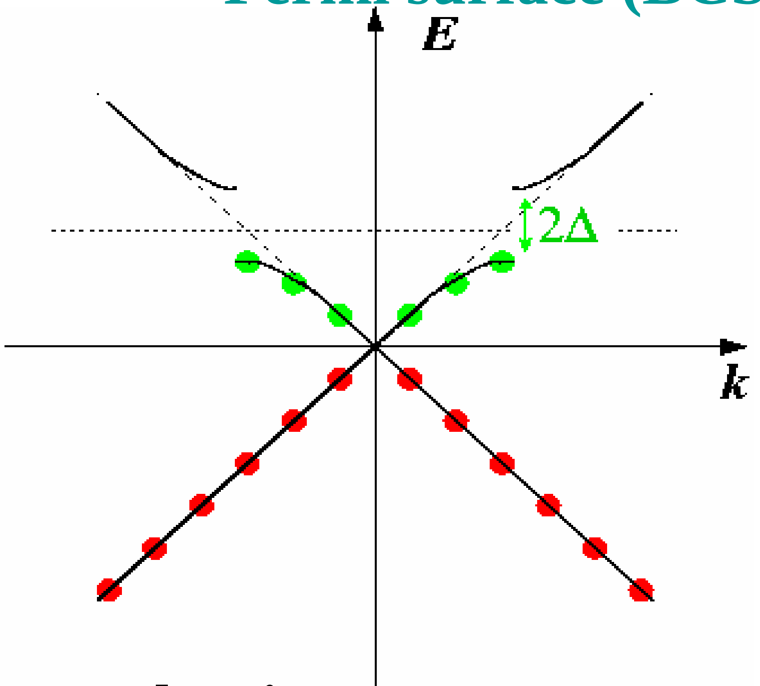


BCS in a degenerated quark matter

small T but large baryon-chemical potential

attractive quark-quark interaction ⇨ **pairing instability of Fermi surface (BCS)**





J. Berges & K. Rajagopal

E. Shuryak, et al..

Large density QCD – Gap equation

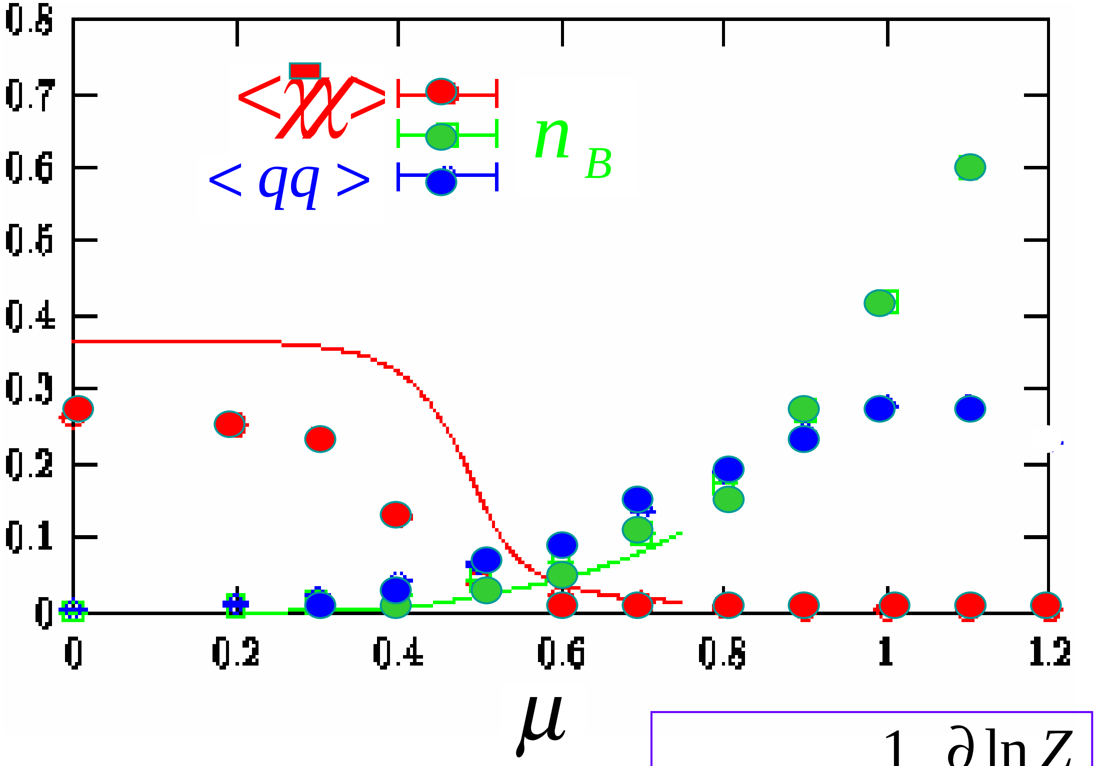
T. Schäfer & F. Wilczek

R. Pisarski & D. Rischke

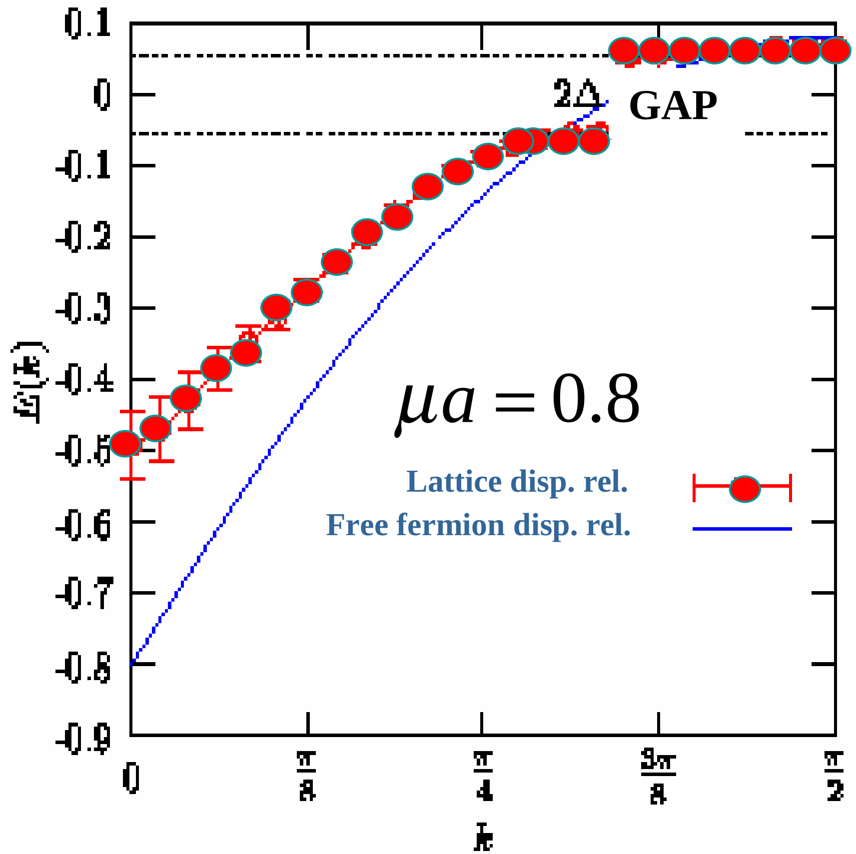
BCS Gap in MC-study of NJL-model

S. Hands & N Walters

$$L = \psi^{tr} M(\mu_q, j)\psi + \frac{2}{g^2}\left(\sigma^2 + \vec{\pi}\times\vec{\pi}\right) \qquad M = \frac{1}{2}\begin{pmatrix} \bar{j}\tau_2 & m \\ -m^{tr} & j\tau_2 \end{pmatrix}$$

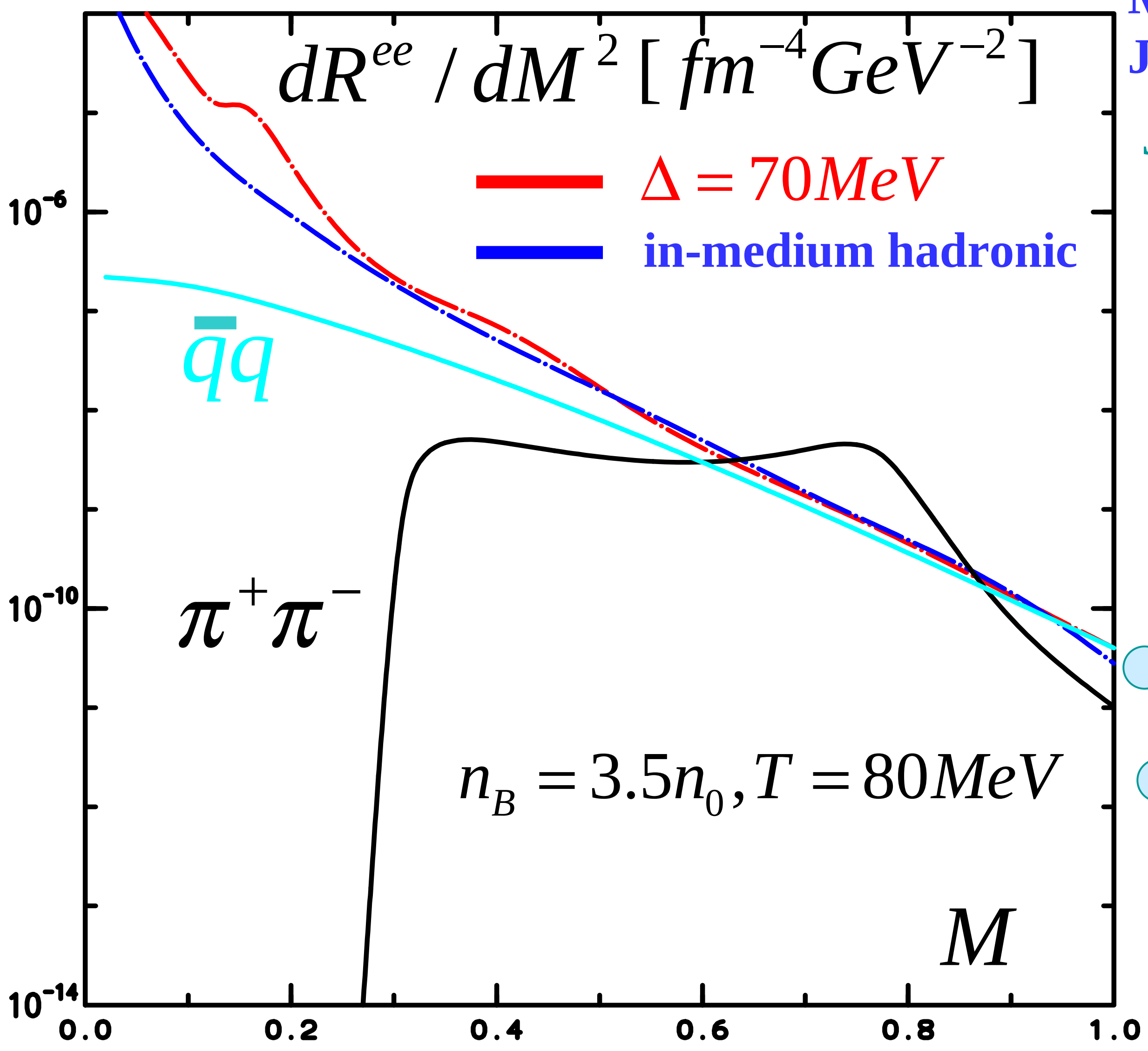


$$<qq> = \frac{1}{2V}\frac{\partial \ln Z}{\partial j}$$



GAP~60-100 MeV

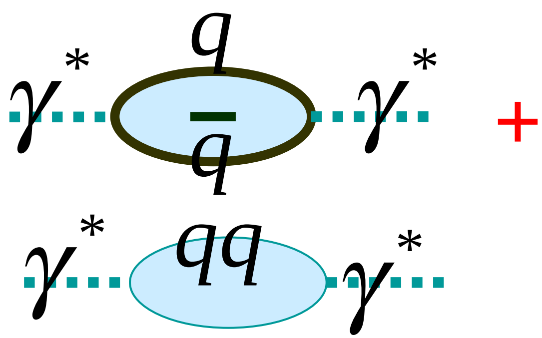
Dileptons from color-flavor locked CFL phase



M. Urban, M. Buuballa, R. Rapp, J. Wambach

J. Cleymans, J. Fingberg, K.R.

P. Jaikumar, R. Rapp, I. Zahed



- **hadron – quark duality**
- **CFL contribution required large gap Δ !**

Conclusions

- Resonances are essential degrees of freedom near deconfinement ⇘ is density driven

  - eqs LGT ⟺ eqs heavy ion phenomenology

- Intermediate energy heavy ion collisions (1-40) AGeV (GSI future experiment) ⇒ laboratory for collective effects and chiral symmetry restoration in high density and temperature baryonic medium

- Color superconductivity could be accessible via dilepton yields – provided that the energy gap is > 100 MeV